Solution Worksheet: Deductive Arguments

Exercise 1:
Consider the argument below, find if this argument is valid
1. \( A \lor (B \land C) \)
   \( B \rightarrow C \)
   \( C \land D \rightarrow F \)
   \( \neg A \)
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\( \therefore F \)
\( \neg A = T \Rightarrow A = F \) (negation)
\( A \lor (B \land C) = T \Rightarrow \)
\( (B \land C) = T \Rightarrow (B = T) \land (C = T) \) (Simplification)
\( \neg A \)
\( (C \land D \rightarrow F) = T \)
\( (T \land D) \rightarrow F = T \)
\( (D \rightarrow F) = T \)
if \( D = False \), \( (D \rightarrow F) = T \)
We cannot conclude that \( F \) is True \( \Rightarrow \) The argument is NOT Valid

Exercise 2:
Translate the following sentences into logical statements using the mentioned predicate functions and quantifiers. Construct a derivation that corresponding to the arguments. If a conclusion does not follow, write CDNF, otherwise show the inference rules used:
Some scientific subjects are not interesting, but all scientific subjects are edifying. Therefore some edifying things are not interesting. \((Sx, Ix, Ex)\).

\( Sx: x \) is a scientific subject, \( Ix: x \) is interesting, \( Ex: x \) is edifying

The Argument:

\[ \exists x \quad (Sx \land \neg Ix) \]
\[ \forall x \quad Sx \Rightarrow Ex \]

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\[ \exists x \quad (Ex \land \neg Ix) \]

Simplification:

\[ \exists x \quad Sx \]
\[ \exists x \quad \neg Ix \]

Universal Instantiation:

\[ \forall x \quad (Sx \Rightarrow Ex) \Rightarrow \]
\[ \exists x \quad Sx \land Ex \]

Simplification:

\[ \exists x \quad Sx \]
\[ \exists x \quad Ex \]

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\[ \exists x \quad (Ex \land \neg Ix) \]

conjunction:

\[ \exists x \quad (Ex \land \neg Ix) \]

VALID
1. All thoughts are short and detached from each other. There is not one of these thoughts which does not contain some great principle or some edifying truth. Therefore all great principles are contained in short thoughts.

Tx: x is a thought
Sx: x is short
Dx: x is detached
Px: x is a great principle
Ex: x is an edifying truth

The argument:

\[ \forall x, \, \text{Tx} \rightarrow \text{Sx} \land \text{Dx} \]

\[ \neg \exists x \, \neg (\text{Tx} \land (\text{Px} \lor \text{Ex})) \]

\[ \therefore \forall x \, (\text{Px} \rightarrow \text{Sx}) \]

Material Implication Truth Table:

<table>
<thead>
<tr>
<th>Sx</th>
<th>Px</th>
<th>Sx v ¬Px</th>
<th>Px → Sx</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
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<tr>
<td>T</td>
<td>F</td>
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<td>T</td>
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</tbody>
</table>

Exercise 3:
Jim, George and Sue belong to an outdoor club. Every club member is either a skier or a mountain climber, but no member is both. No mountain climber likes rain, and all skiers like snow. George dislikes whatever Jim likes and likes whatever Sue dislikes. Jim and Sue both like rain and snow. Is there a member of the outdoor club who is a mountain climber?

Ox: x is a member of an outdoor club
Sx: x is a skier
Mx: x is a mountain climber
L(x, y): x likes y,
G= George, J= Jim, S= Sue

1-∀ x, Ox → (Sx ^ ¬Mx) v (Mx ^ ¬Sx)
2- ¬∃ x  Mx ^ L(x, Rain)
3- ∀ x  Sx→ L(x, Snow)
4- ∀ y L (J, y) → ¬ L(G, y)
5- ∀ y ¬ L (S, y) → L(G, y)
6- L (S, Snow) ^ L (S, Rain)
7- L(J, Snow) ^ L(S, Snow)
8- O_G: George is a an outdoor club member
9- O_S: Sue is an outdoor club member
10- O_J: Jim is an outdoor club member

************************************************************************
:: ∃ x  Mx ^ Ox?

4 & 7 (modus ponens) ⇒ 4’ ¬L(G, Snow) ^ ¬L(G, Rain).
3 (contrapositive) ⇒ (3’)
∀ x ¬L(x, Snow) → ¬Sx
3’ & 4’ (universal instantiation) ⇒ ¬S_G = T : George is not a skier
1 Universal instantiation:
1’ O_G → (S_G ^ ¬M_G) v (M_G ^ ¬S_G)
¬S_G=T ⇒ S_G=F ⇒
1’ & 8⇒ O_G → M_G ^ O_G
⇒ M_G (modus ponens) ⇒ (Existential generalization) ⇒ ∃ x Mx ^ Ox

VALID