Trees and Search Strategies and Algorithms --

Reference: Dr. Franz J. Kurfess

Computer Science Department

Cal Poly

Basic Search Strategies

- depth-first
- breadth-first

- exercise
  - apply depth-first to finding a path from this building to your favorite “feeding station” (McDonalds, Jason Deli, Pizza Hut)
    - is this task sufficiently specified
    - is success guaranteed
    - how long will it take
    - could you remember the path
    - how good is the solution

Motivation

Motivation

- search strategies are important methods for many approaches to problem-solving
- the use of search requires an abstract formulation of the problem and the available steps to construct solutions
- search algorithms are the basis for many optimization and planning methods

Objectives

- formulate appropriate problems as search tasks
  - states, initial state, goal state, successor functions (operators), cost
- know the fundamental search strategies and algorithms
    - breadth-first, depth-first,
- evaluate the suitability of a search strategy for a problem
  - completeness, time & space complexity, optimality

Problems

- solution
  - path from the initial state to a goal state
- search cost
  - time and memory required to calculate a solution
- path cost
  - determines the expenses of the agent for executing the actions in a path
  - sum of the costs of the individual actions in a path
- total cost
  - sum of search cost and path cost
  - overall cost for finding a solution

Traveling Salesperson

- states
  - locations / cities
  - illegal states
    - each city may be visited only once
    - visited cities must be kept as state information
- initial state
  - starting point
  - no cities visited
- successor function (operators)
  - move from one location to another one
- goal test
  - all locations visited
  - agent at the initial location
- path cost: distance between locations

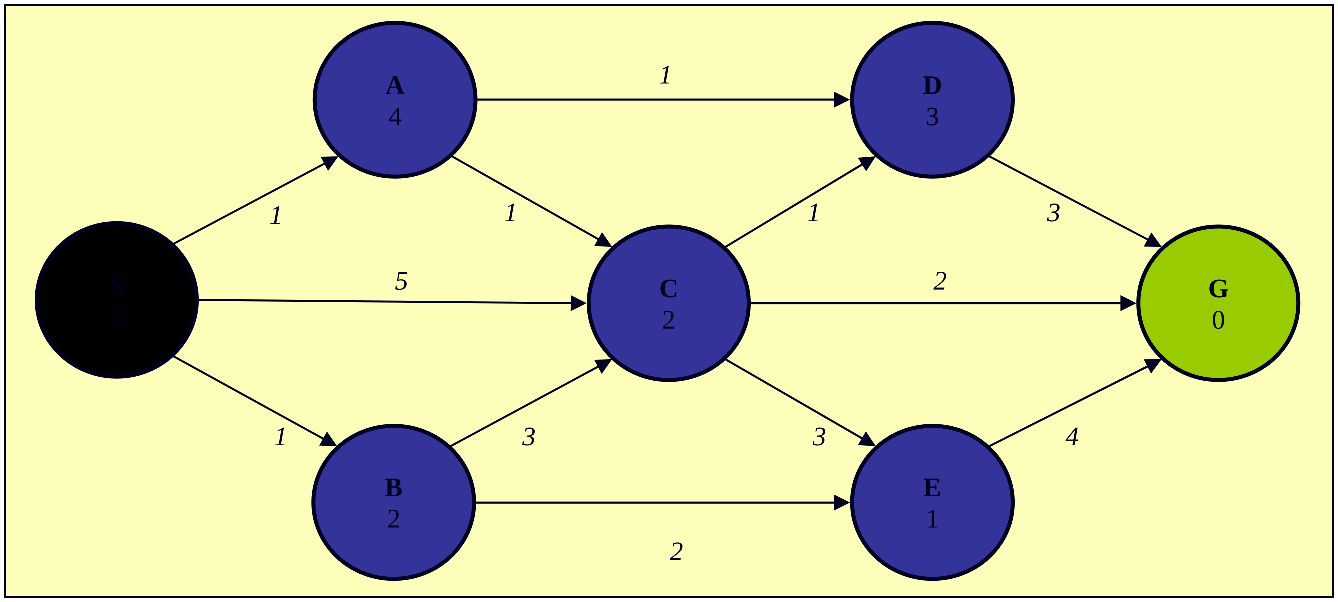
Searching for Solutions

- traversal of the search space
  - from the initial state to a goal state
  - legal sequence of actions as defined by successor function (operators)
- general procedure
  - check for goal state
  - expand the current state
    - determine the set of reachable states
    - return "failure" if the set is empty
  - select one from the set of reachable states
  - move to the selected state
- a search tree is generated

Search Terminology

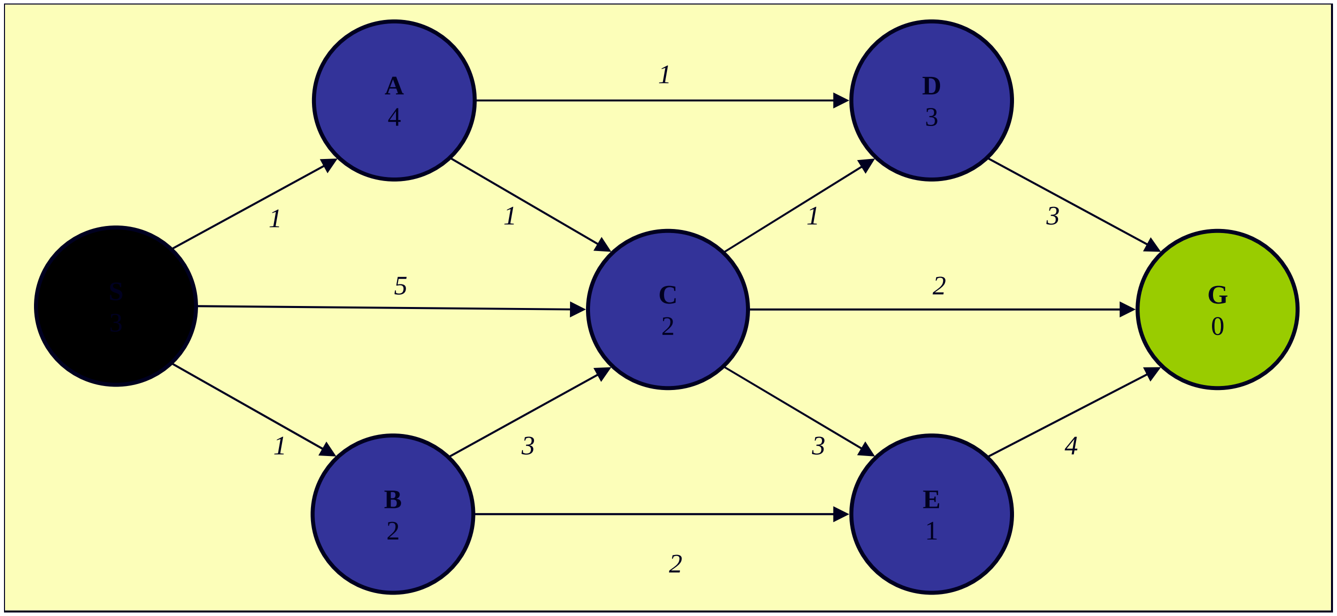
- search tree
  - generated as the search space is traversed
    - the search space itself is not necessarily a tree, frequently it is a graph
    - the tree specifies possible paths through the search space
  - expansion of nodes
    - as states are explored, the corresponding nodes are *expanded* by applying the successor function
      - this generates a new set of (child) nodes
    - the *fringe* (frontier) is the set of nodes not yet visited
      - newly generated nodes are added to the fringe
  - search strategy
    - determines the selection of the next node to be expanded
    - can be achieved by ordering the nodes in the fringe
      - e.g. queue (FIFO), stack (LIFO), “best” node w.r.t. some measure (cost)

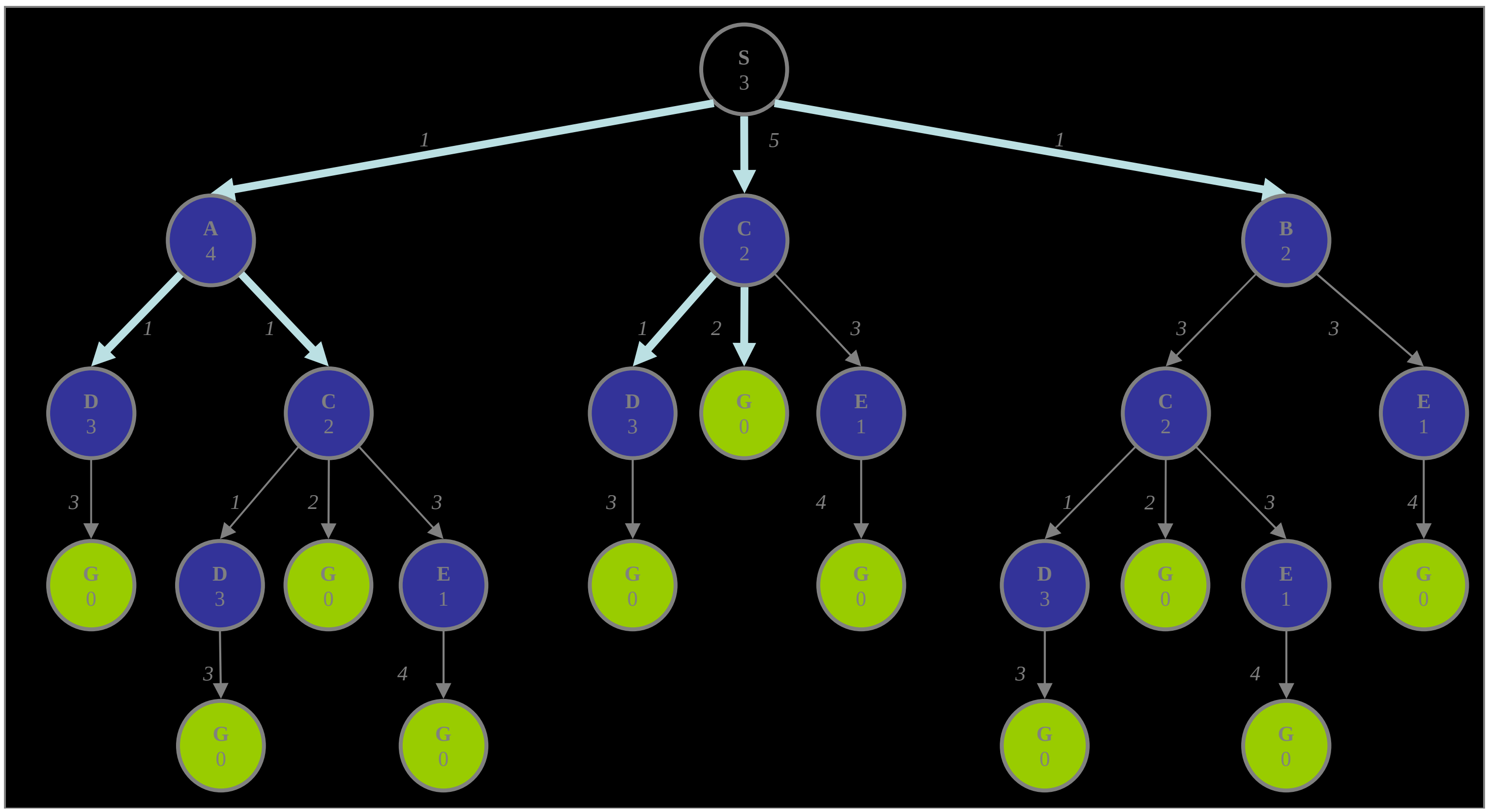
Example: Graph Search

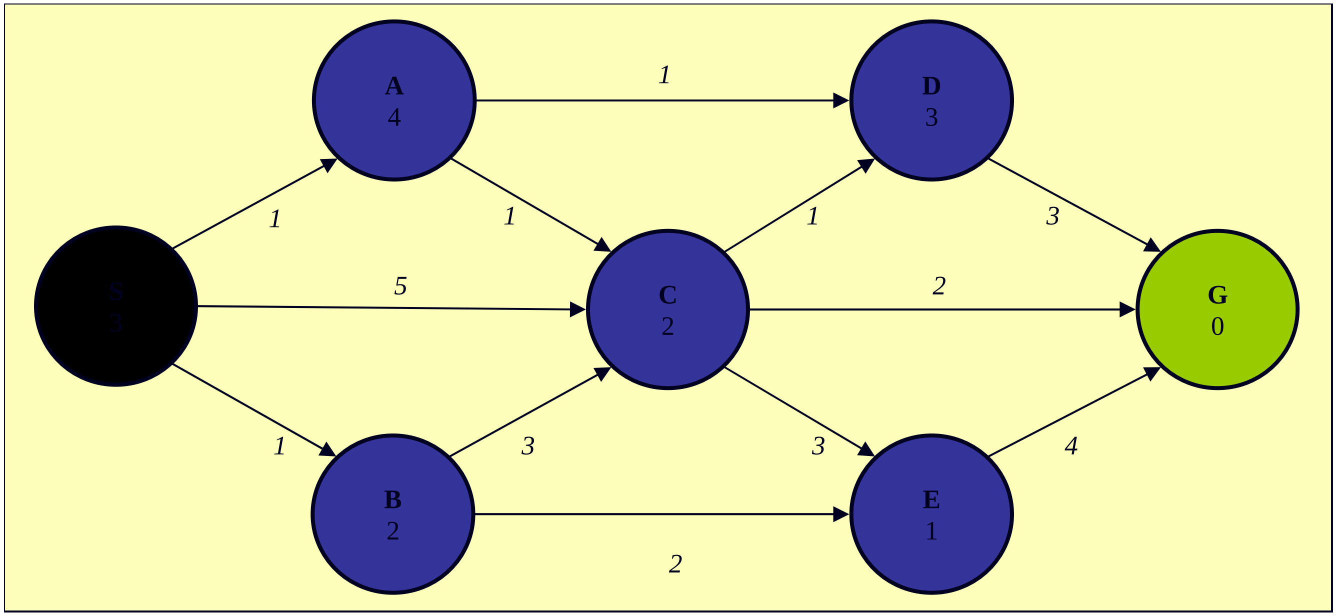


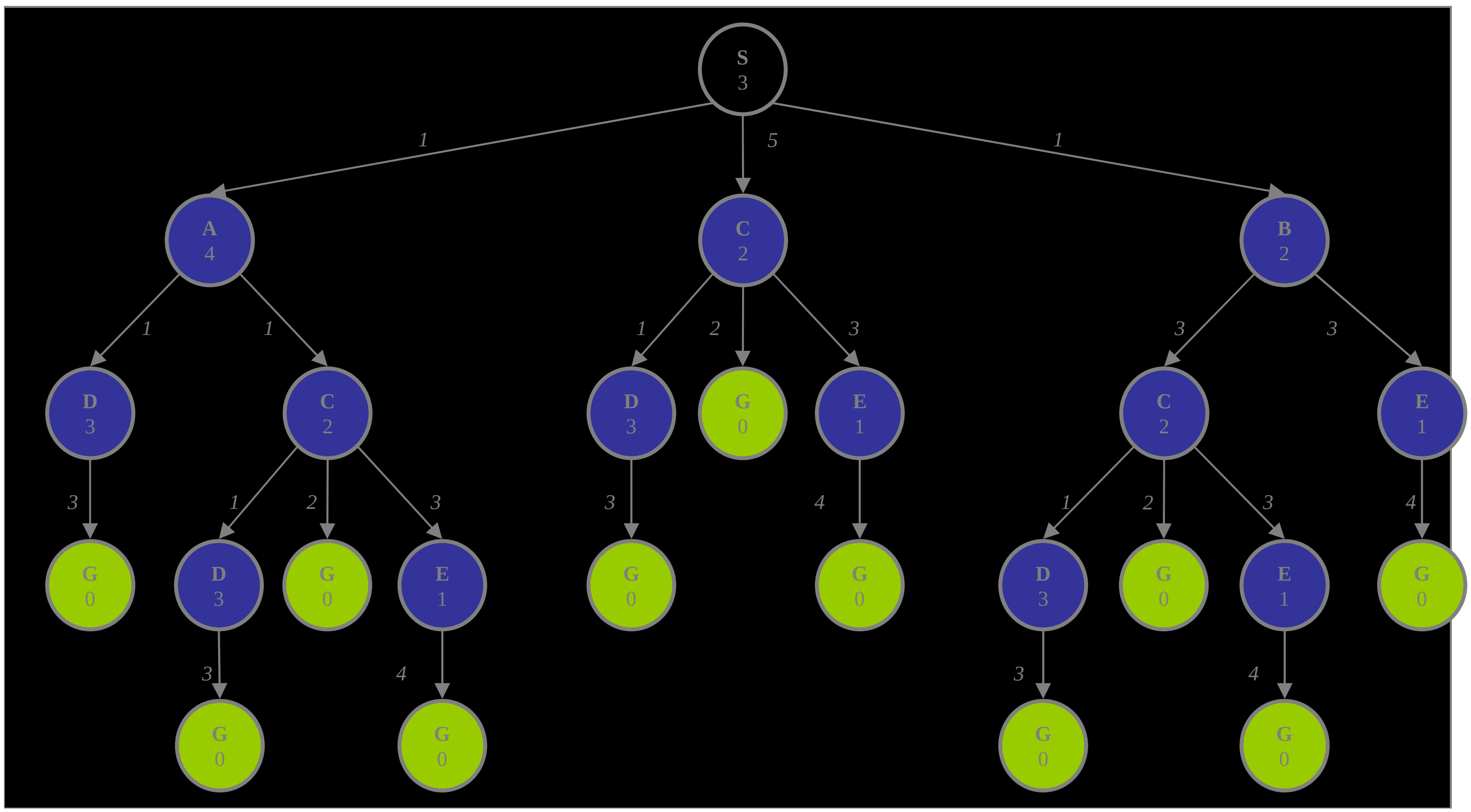
- the graph describes the search (state) space
  - each node in the graph represents one state in the search space. **e.g. a city to be visited in a routing or touring problem**
- this graph has additional information
  - names and properties for the states (e.g. **S**, 3)
  - links between nodes, specified by the successor function
    - properties for links (distance, cost, name, ...)

Breadth First Search



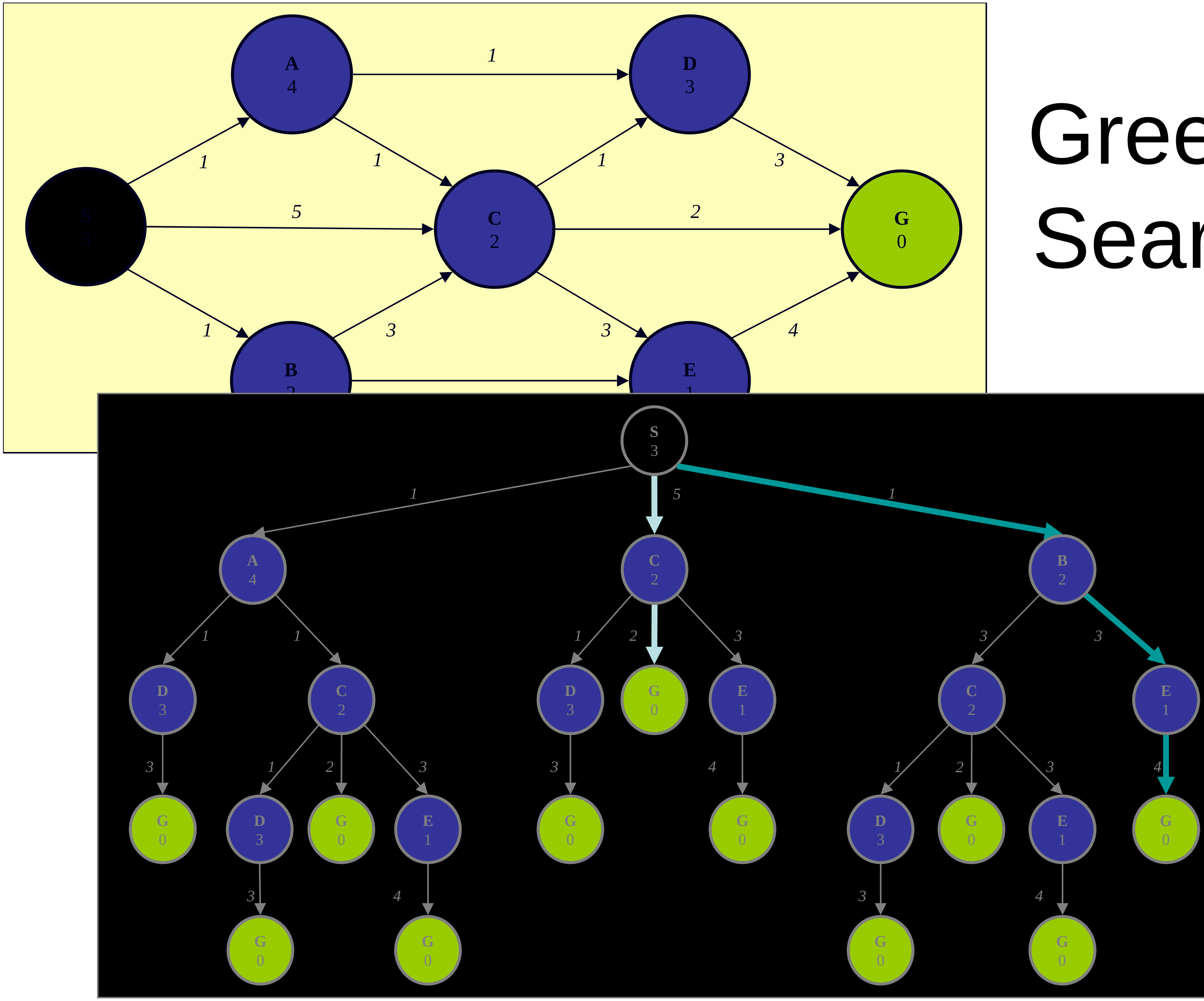




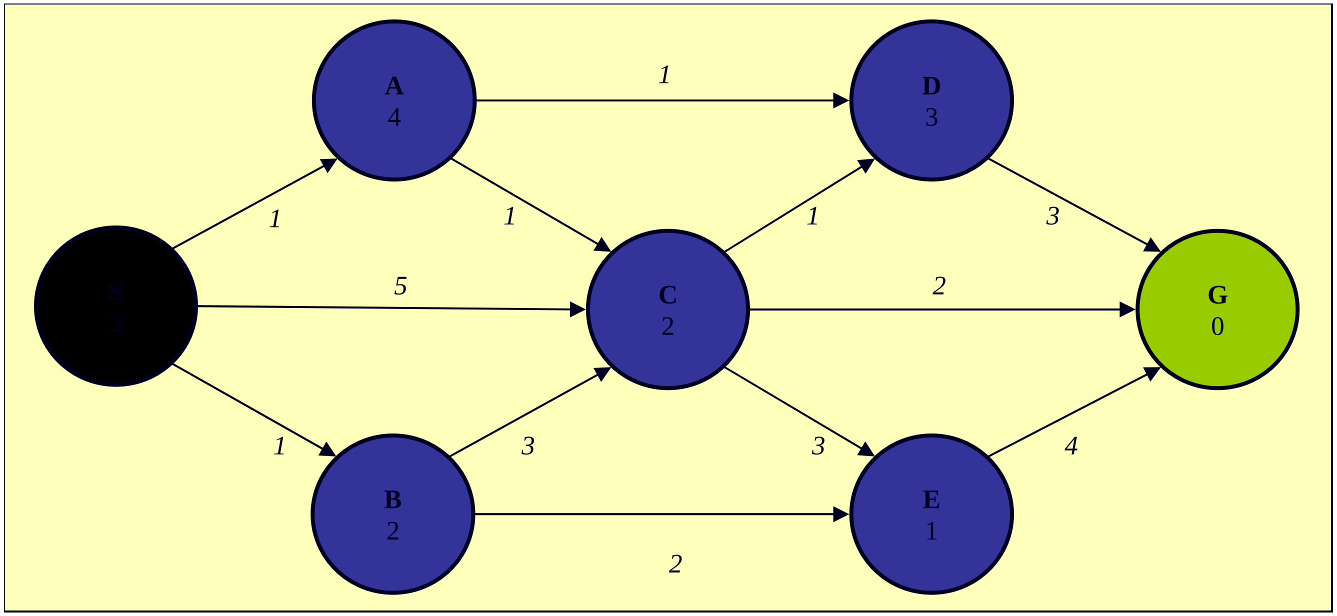


Graph and Tree

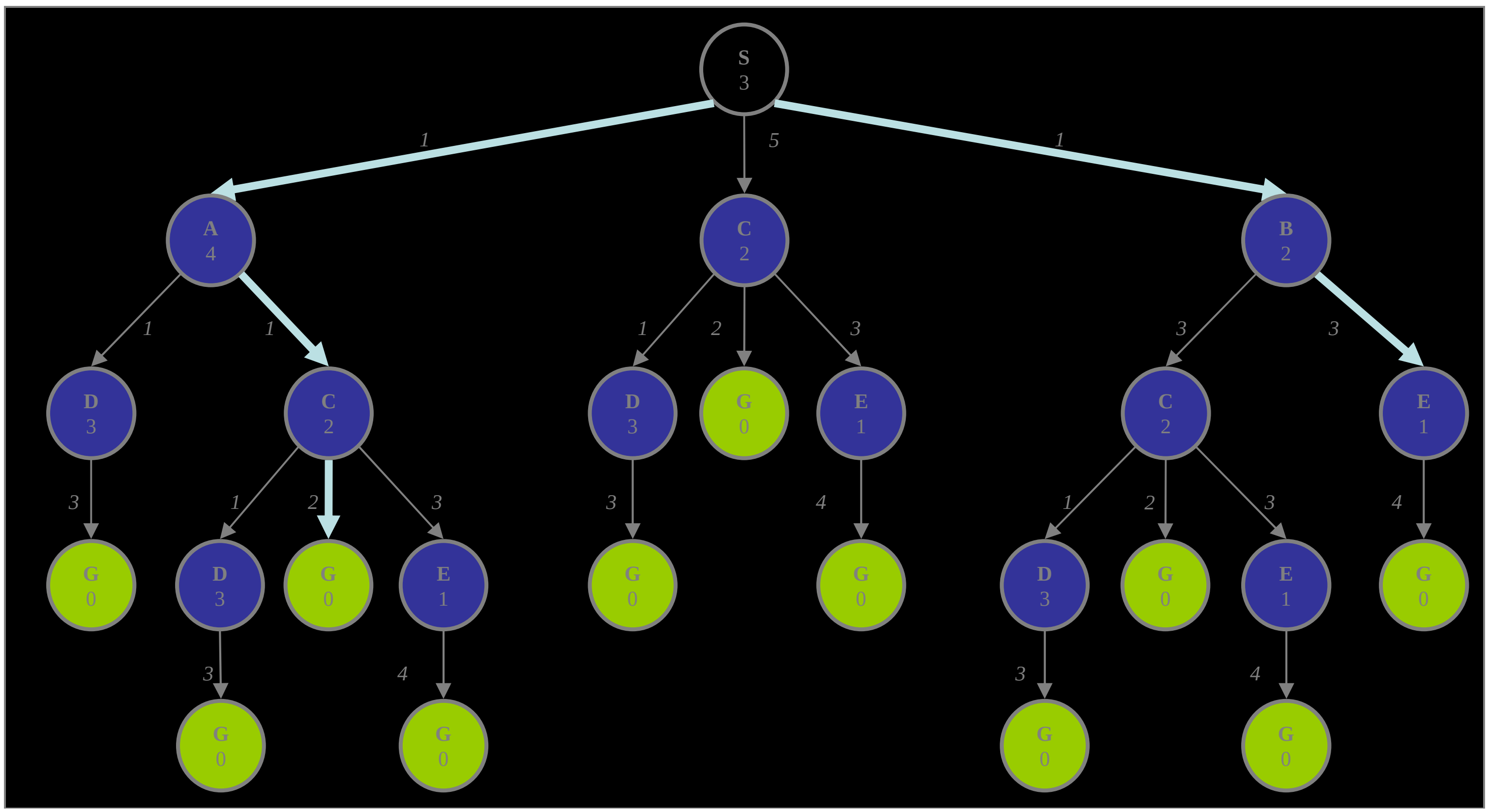
- the tree is generated by traversing the graph
- the same node in the graph may appear repeatedly in the tree
- the arrangement of the tree depends on the traversal strategy (search method)
- the initial state becomes the root node of the tree
- in the fully expanded tree, the goal states are the leaf nodes



Greedy Search


A
4
D
3
C
2
G
0
B
2
E
1
1
1
1
1
3
5
2
1
3
3
4
2

A* Search


S
3
1
5
1
A
4
C
2
B
2
D
3
C
2
D
3
G
0
E
1
C
2
E
1
G
0
D
3
G
0
E
1
G
0
G
0
D
3
G
0
E
1
G
0
G
0
G
0
G
0
G
0

General Search Algorithm

```
function TREE-SEARCH(problem, fringe) returns solution
   fringe := INSERT(MAKE-NODE(INITIAL-STATE[problem]), fringe)
   loop do
     if EMPTY?(fringe) then return failure
     node := REMOVE-FIRST(fringe)
     if GOAL-TEST[problem] applied to STATE[node] succeeds
       then return SOLUTION(node)
     fringe := INSERT-ALL(EXPAND(node, problem), fringe)
```

- generate the node from the initial state of the problem
- repeat
  - return failure if there are no more nodes in the fringe
  - examine the current node; if it’s a goal, return the solution
  - expand the current node, and add the new nodes to the fringe

General Search Algorithm

```
function GENERAL-SEARCH(problem, QUEUING-FN) returns solution
   nodes := MAKE-QUEUE(MAKE-NODE(INITIAL-STATE[problem]))
    loop do
     if nodes is empty then return failure
       node        := REMOVE-FRONT(nodes)
     if GOAL-TEST[problem] applied to STATE(node) succeeds
       then return node
     nodes        := QUEUING-FN(nodes, EXPAND(node,
                     OPERATORS[problem]))
  end
```

Evaluation Criteria

- completeness
  - if there is a solution, will it be found
- time complexity
  - how long does it take to find the solution
  - does not include the time to perform actions
- space complexity
  - memory required for the search
- optimality
  - will the best solution be found

main factors for complexity considerations:

branching factor b, depth d of the shallowest goal node, maximum path length m

Search Cost

- the *search cost* indicates how expensive it is to generate a solution
  - time complexity (e.g. number of nodes generated) is usually the main factor
  - sometimes space complexity (memory usage) is considered as well
- *path cost* indicates how expensive it is to execute the solution found in the search
  - distinct from the search cost, but often related
- *total cost* is the sum of search and path costs

Breadth-First

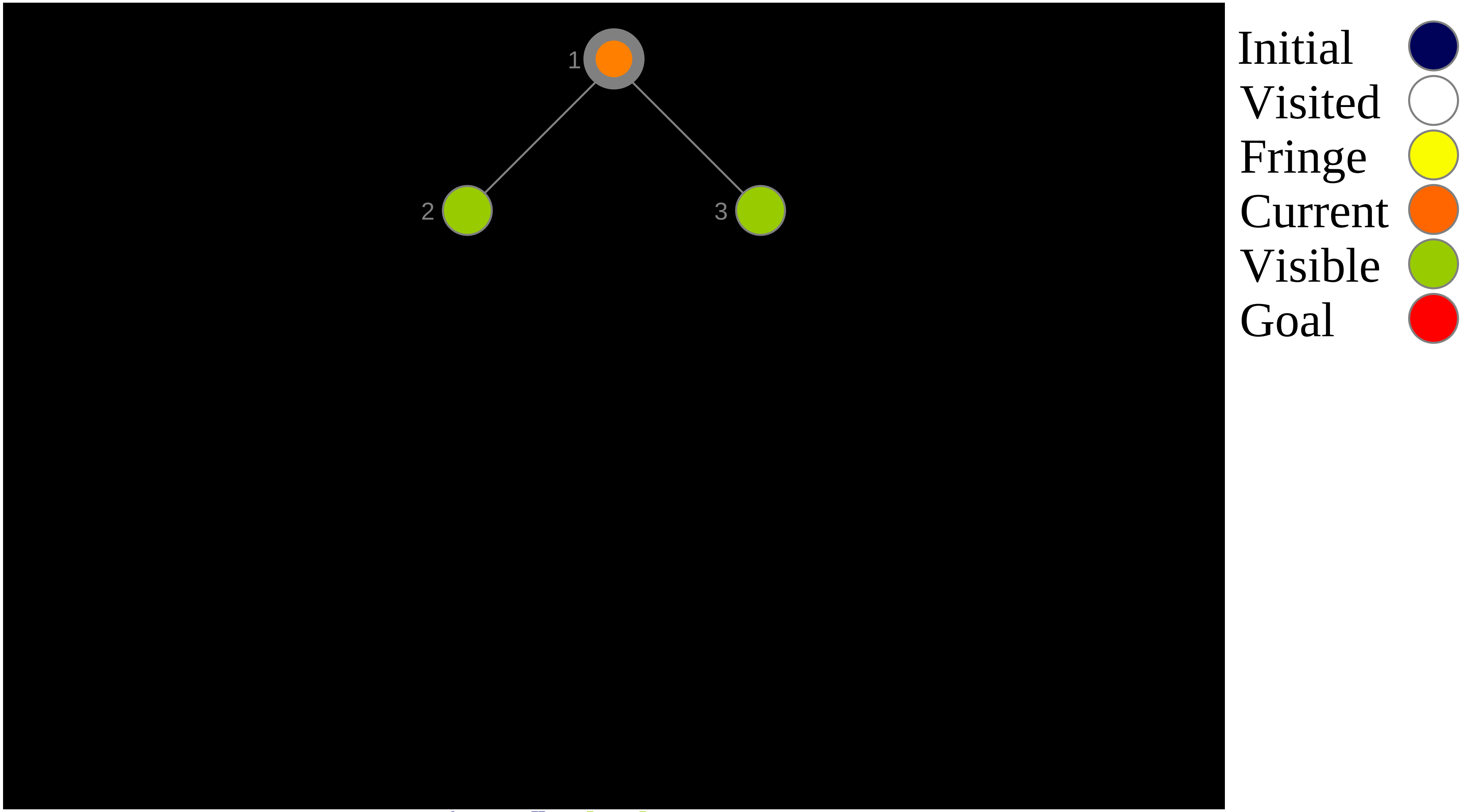
- all the nodes reachable from the current node are explored first
  - achieved by the TREE-SEARCH method by appending newly generated nodes at the end of the search queue

```
function BREADTH-FIRST-SEARCH(problem) returns solution

        return TREE-SEARCH(problem, FIFO-QUEUE())
```

Time Complexity	b^{d+1}
Space Complexity	b^{d+1}
Completeness	yes (for finite b)
Optimality	yes (for non-negative path costs)

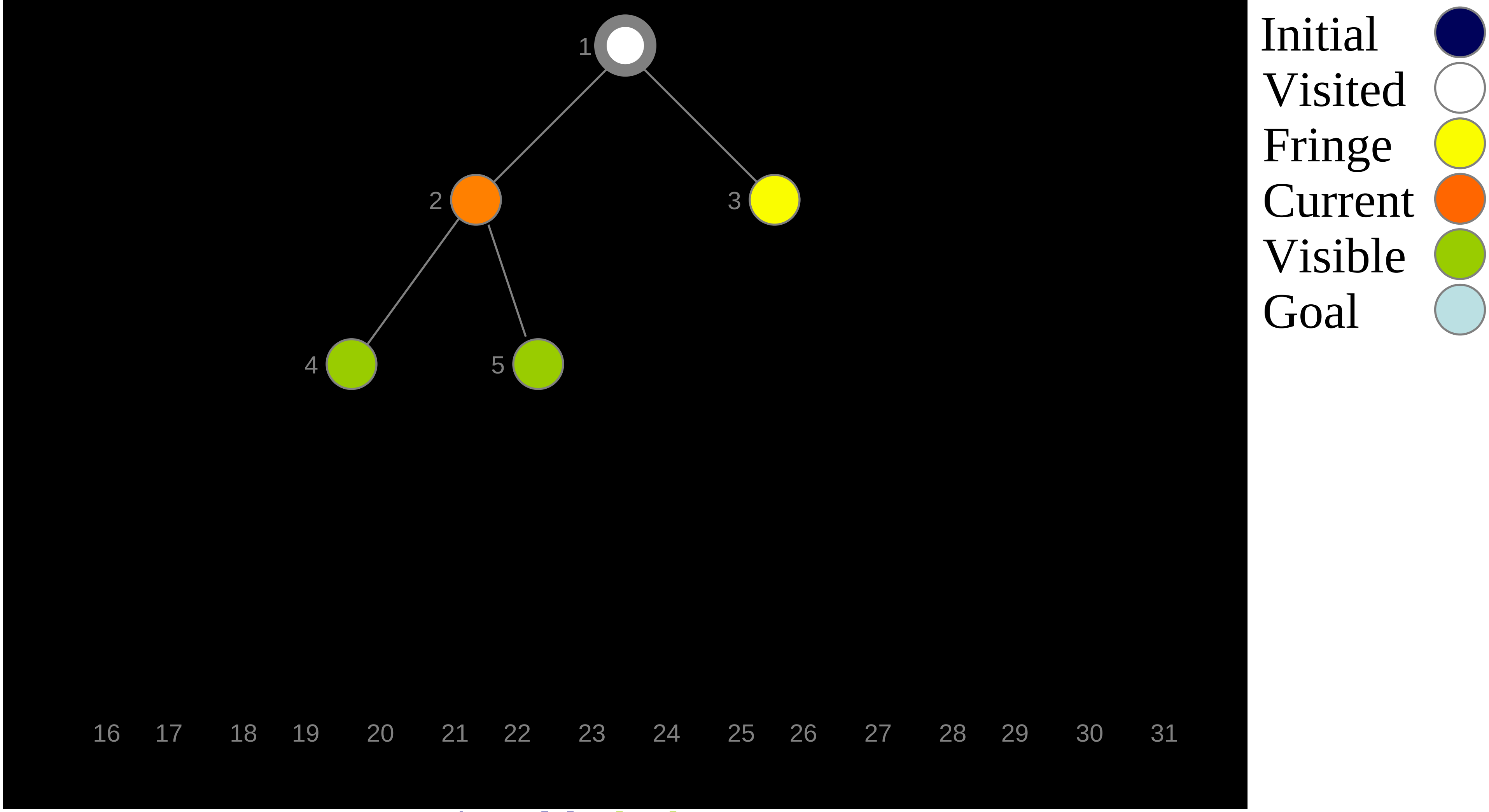
b	branching factor
d	depth of the tree

Breadth-First Snapshot 1



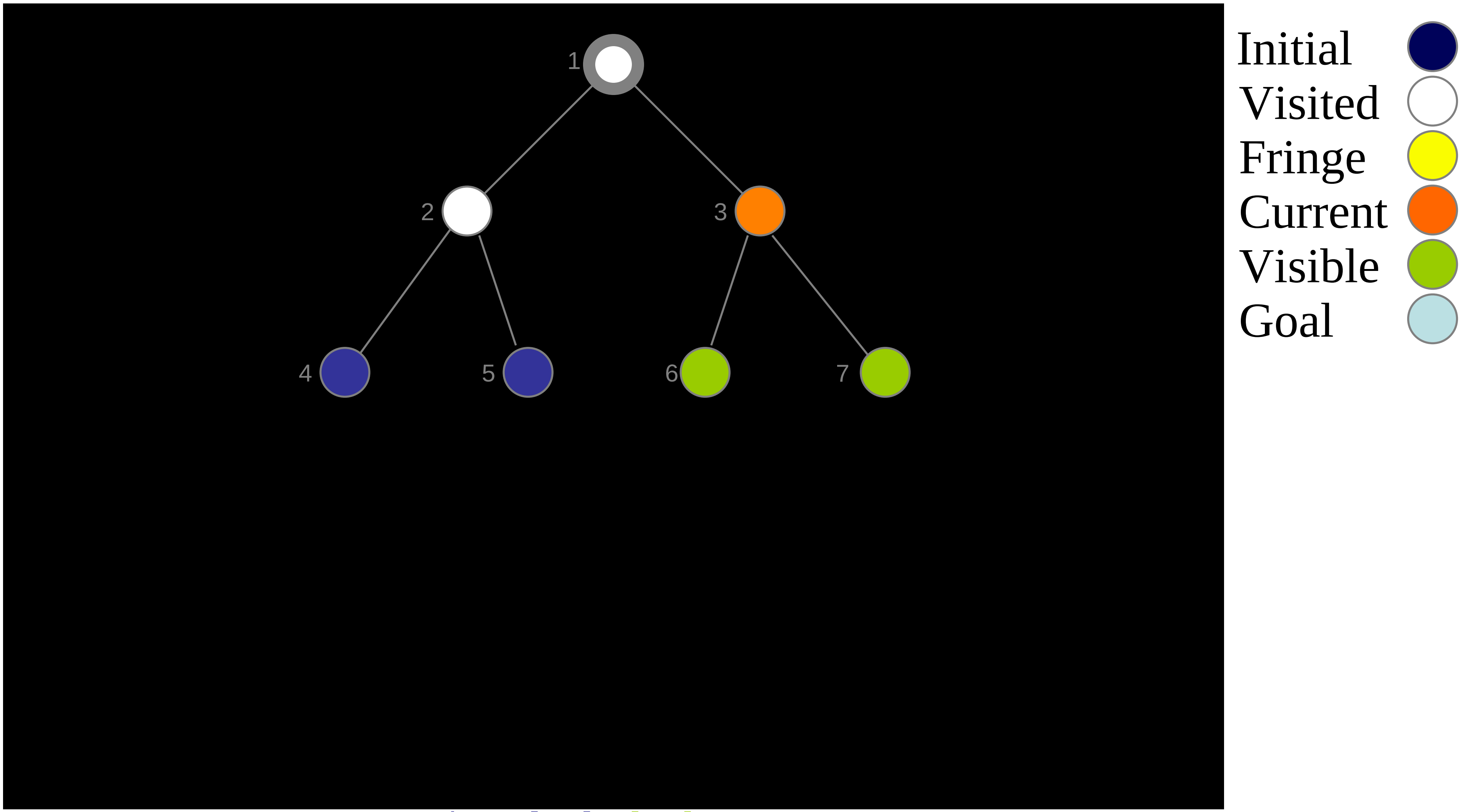
Fringe: [] + [2,3]

Breadth-First Snapshot 2



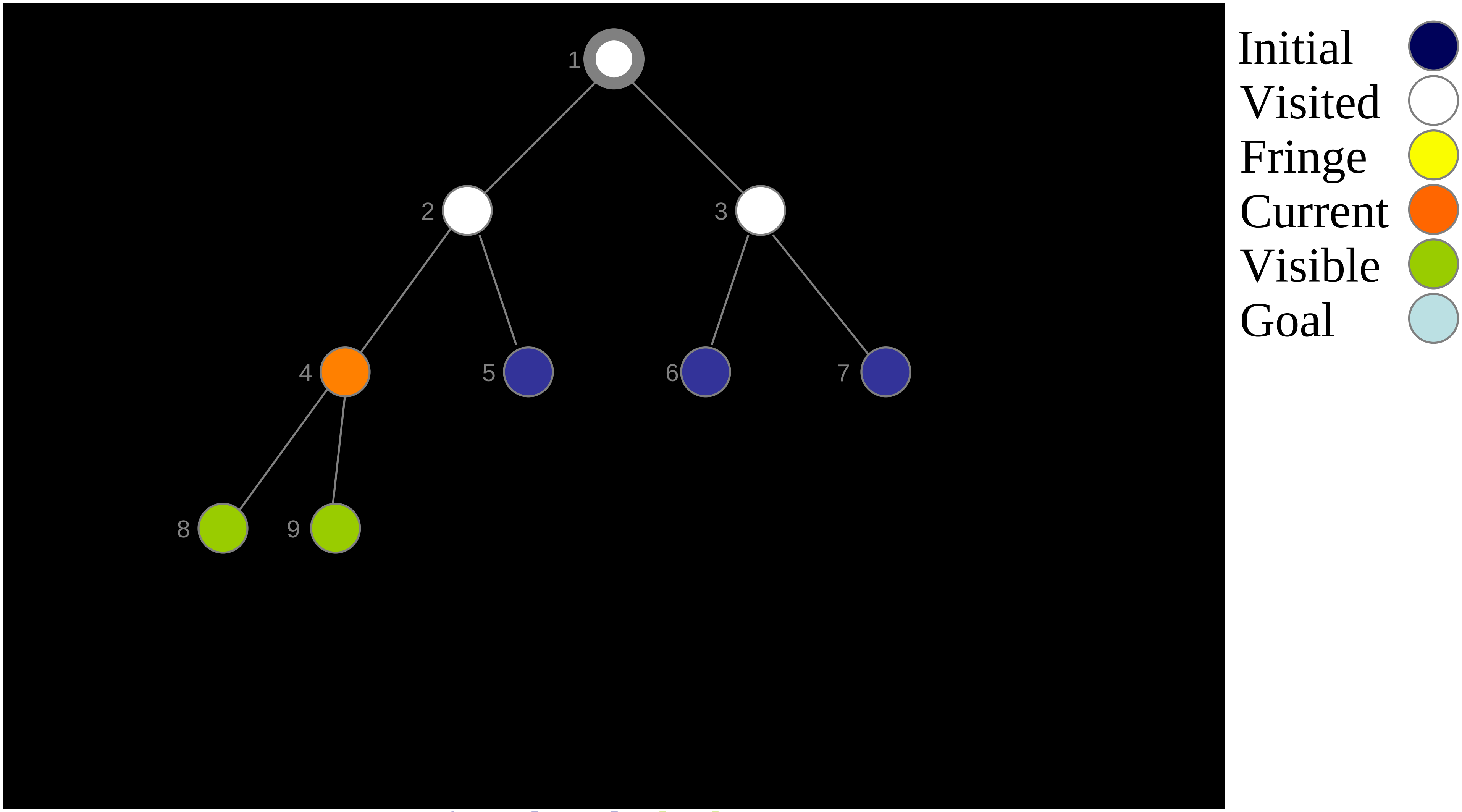
Fringe: [3] + [4,5]

Breadth-First Snapshot 3



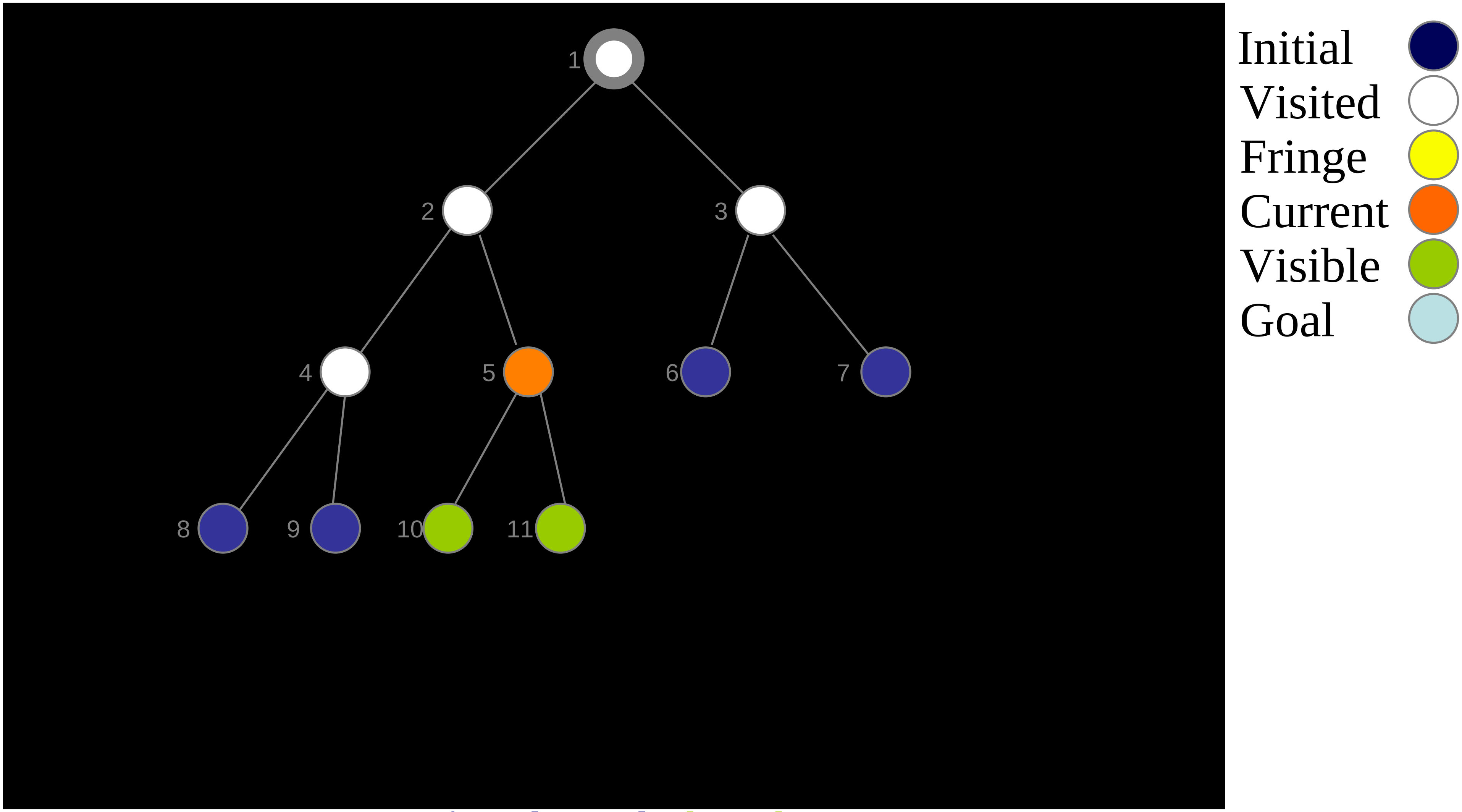
Fringe: [4,5] + [6,7]

Breadth-First Snapshot 4



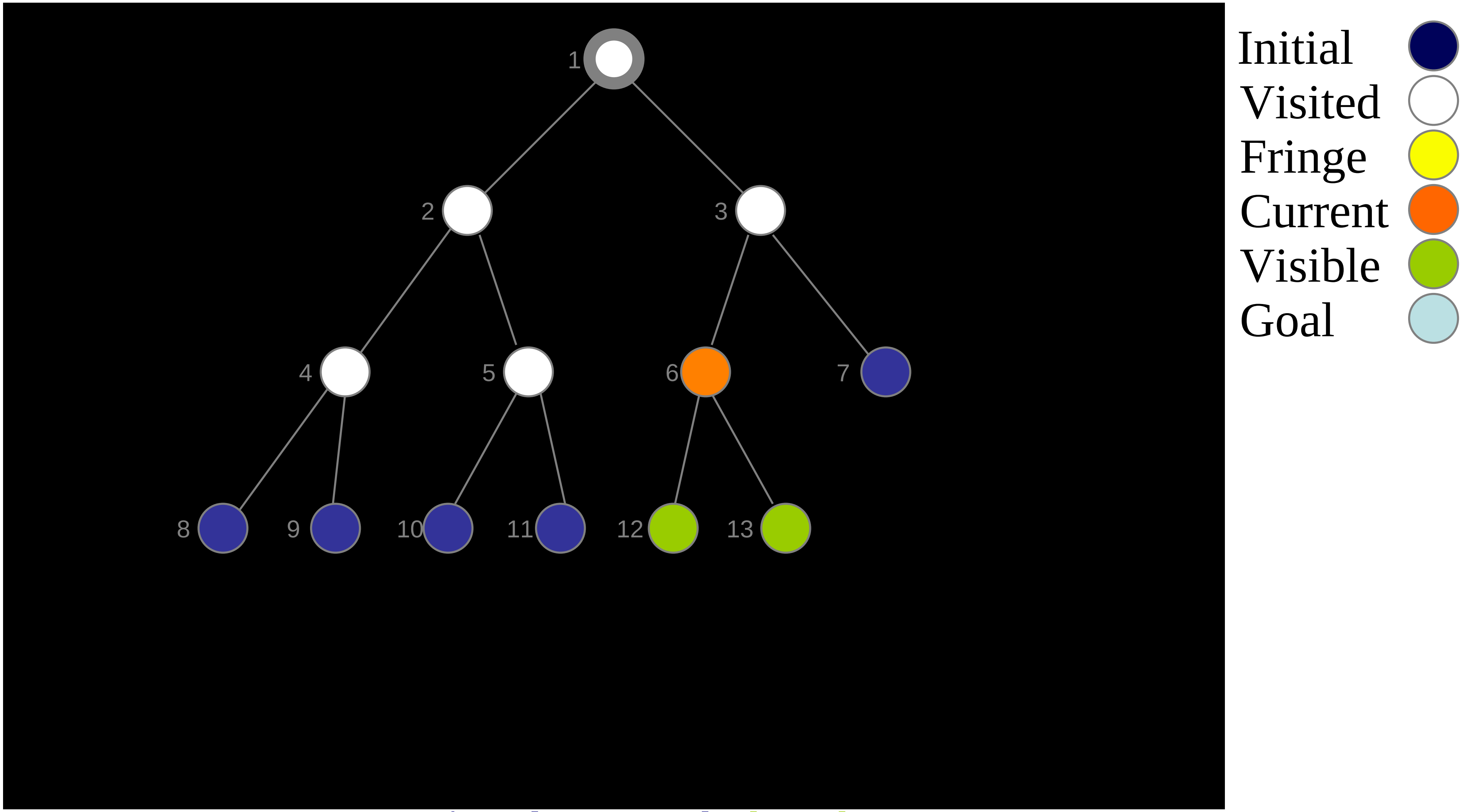
Fringe: [5,6,7] + [8,9]

Breadth-First Snapshot 5



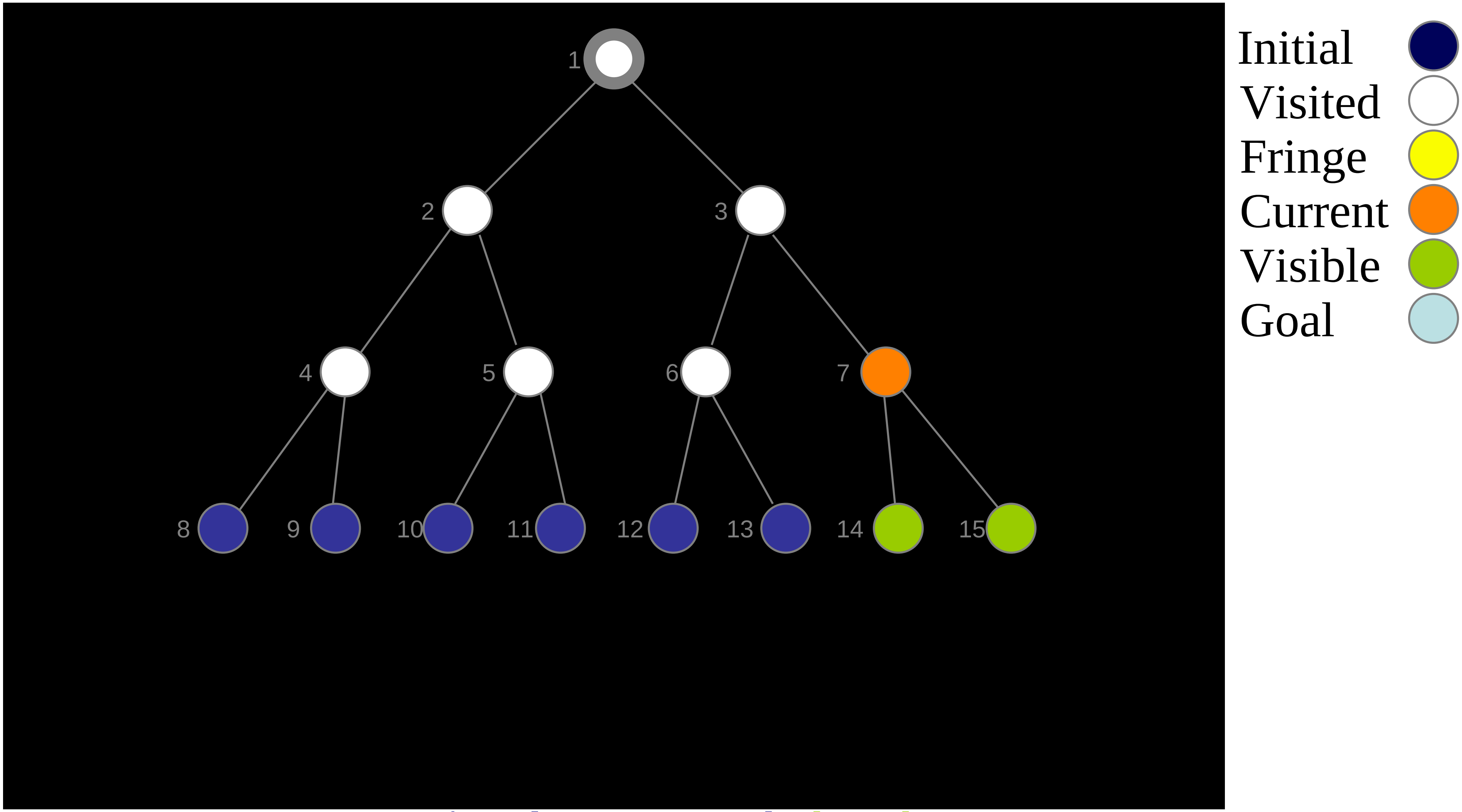
Fringe: [6,7,8,9] + [10,11]

Breadth-First Snapshot 6



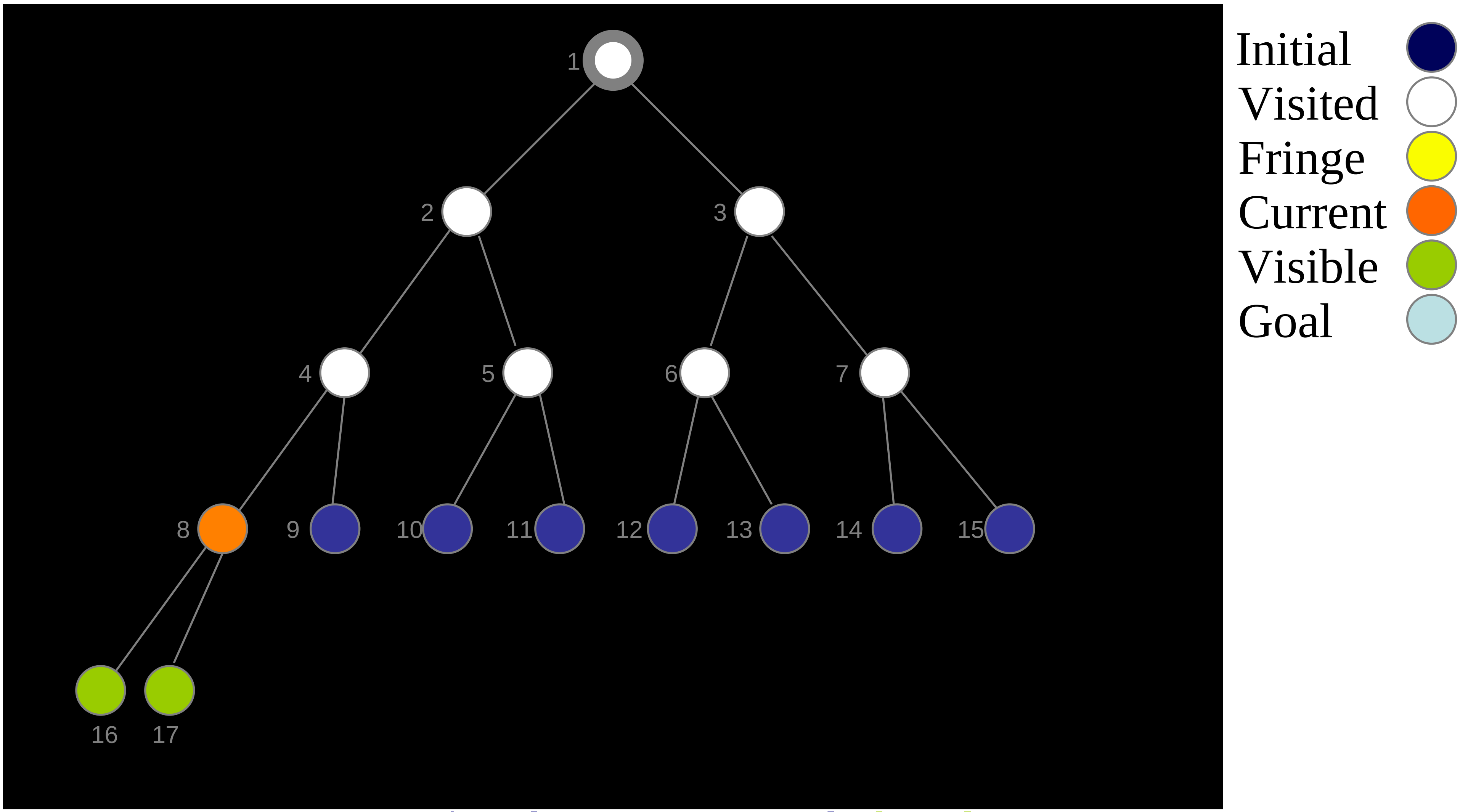
Fringe: [7,8,9,10,11] + [12,13]

Breadth-First Snapshot 7



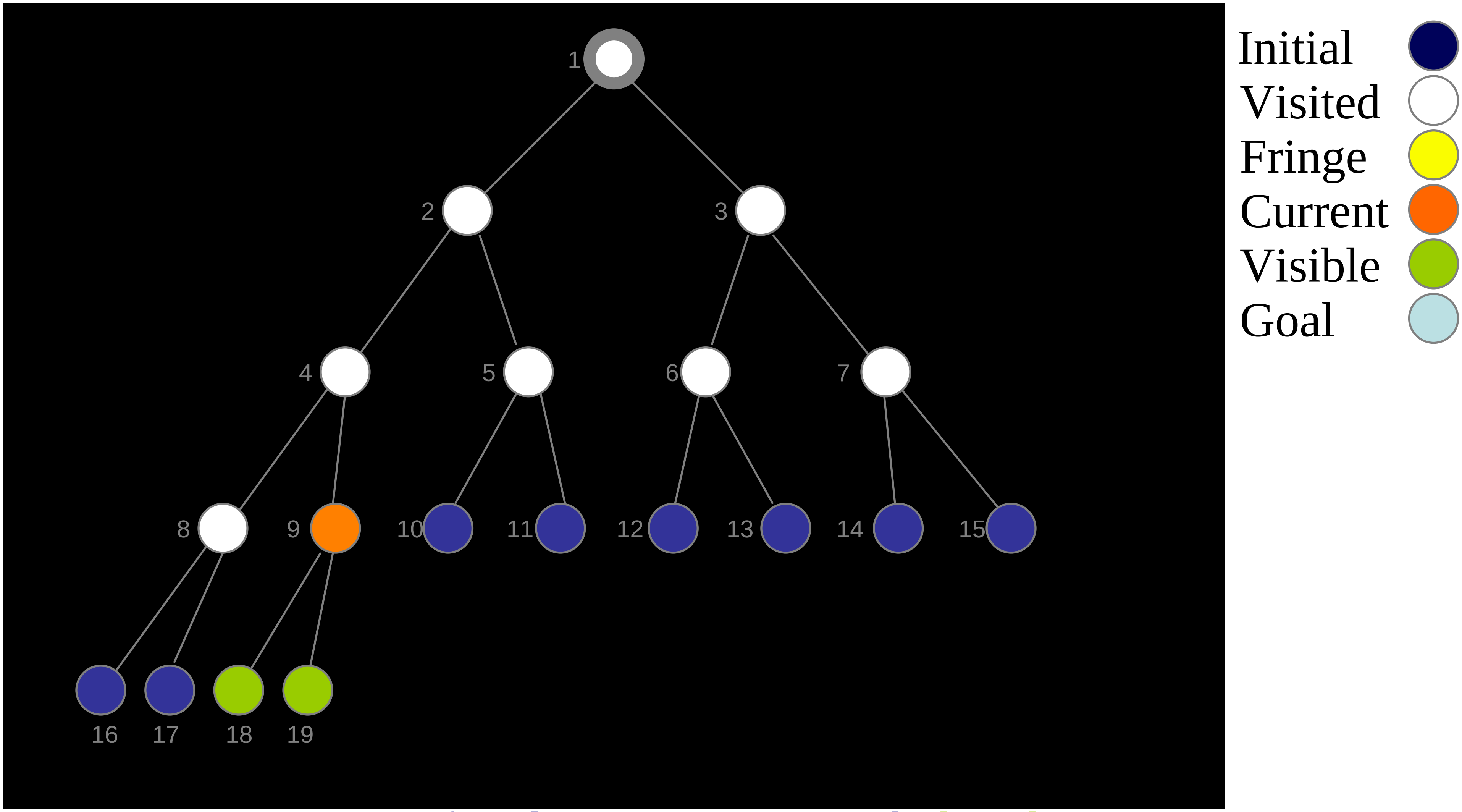
Fringe: [8,9.10,11,12,13] + [14,15]

Breadth-First Snapshot 8



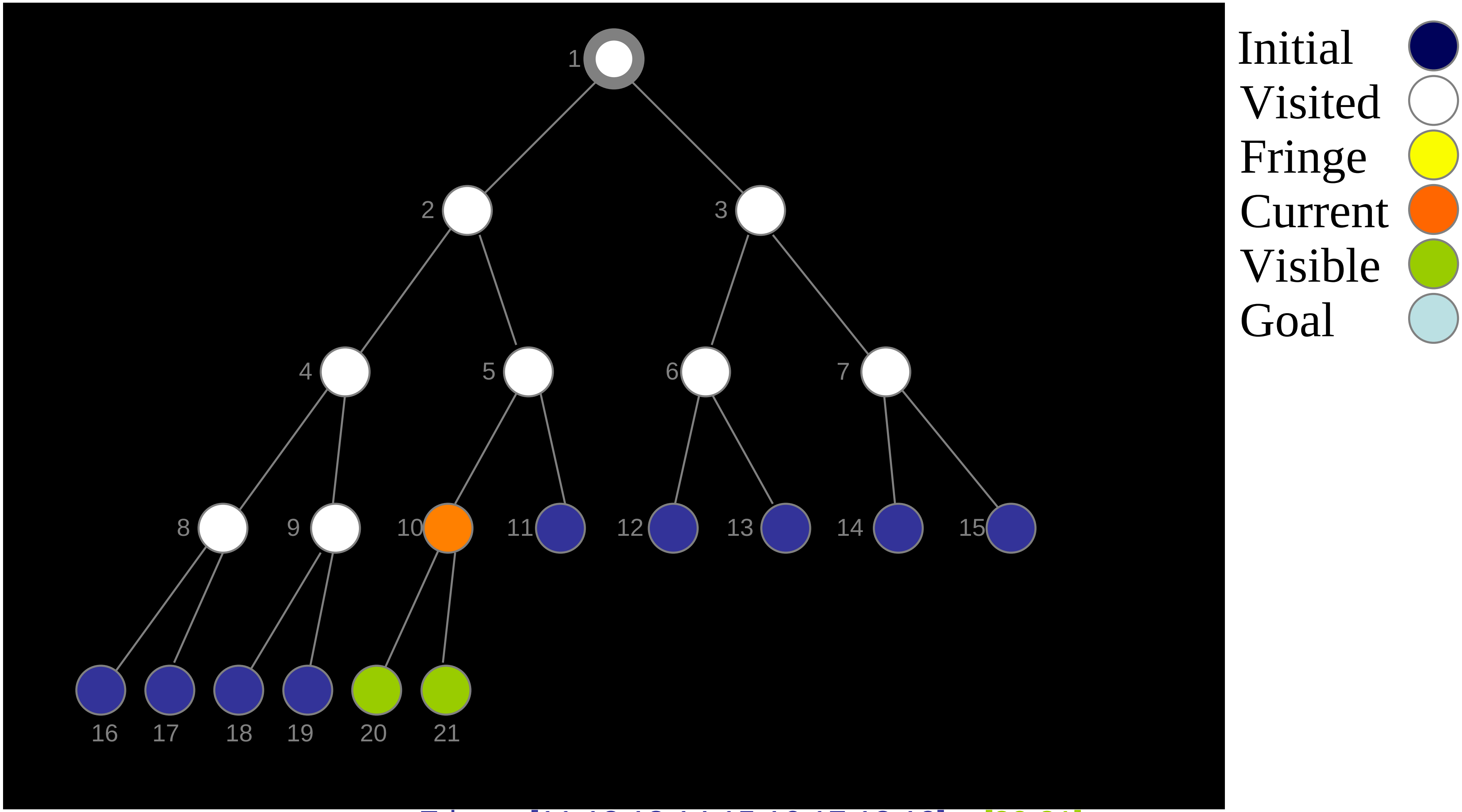
Fringe: [9,10,11,12,13,14,15] + [16,17]

Breadth-First Snapshot 9



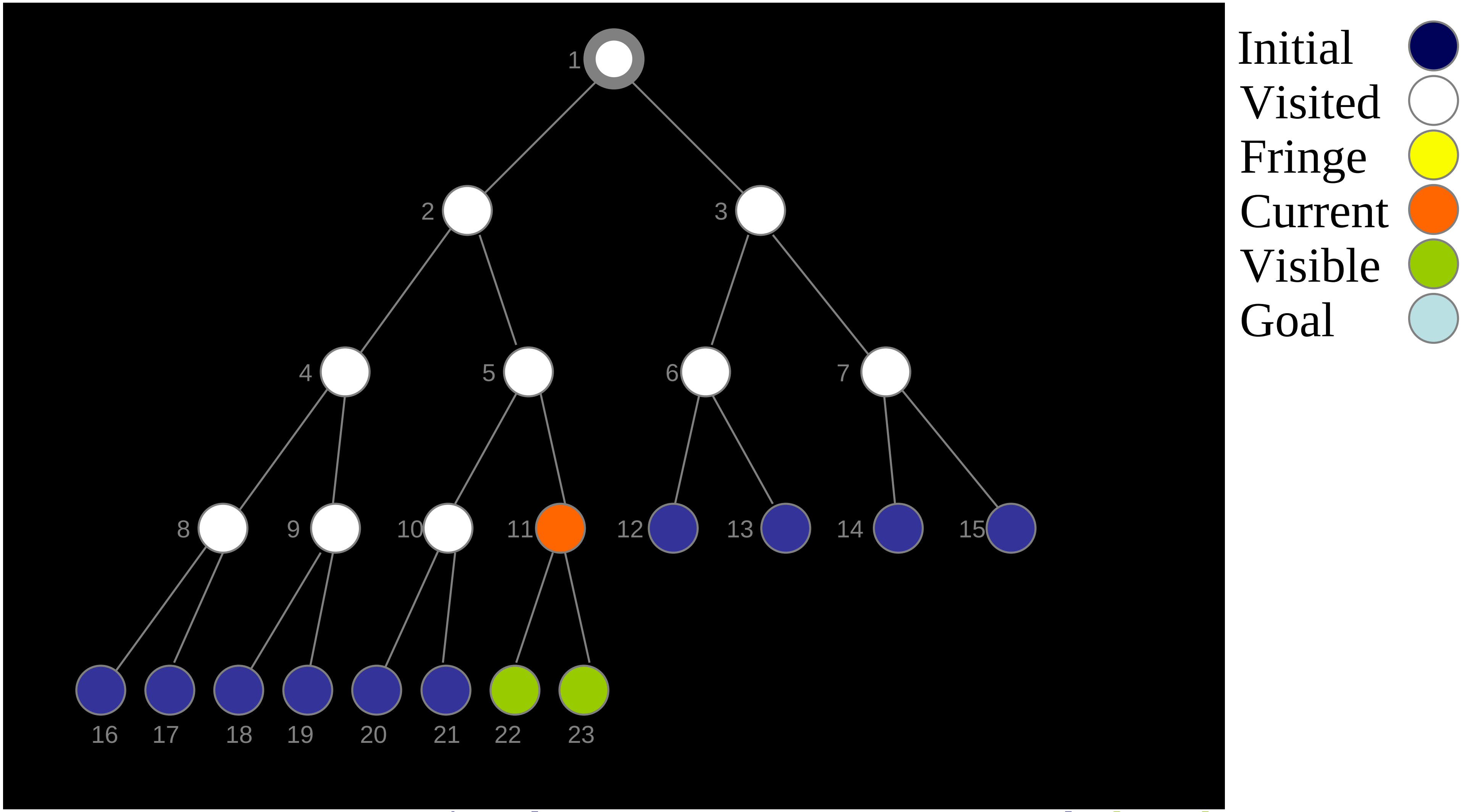
Fringe: [10,11,12,13,14,15,16,17] + [18,19]

Breadth-First Snapshot 10



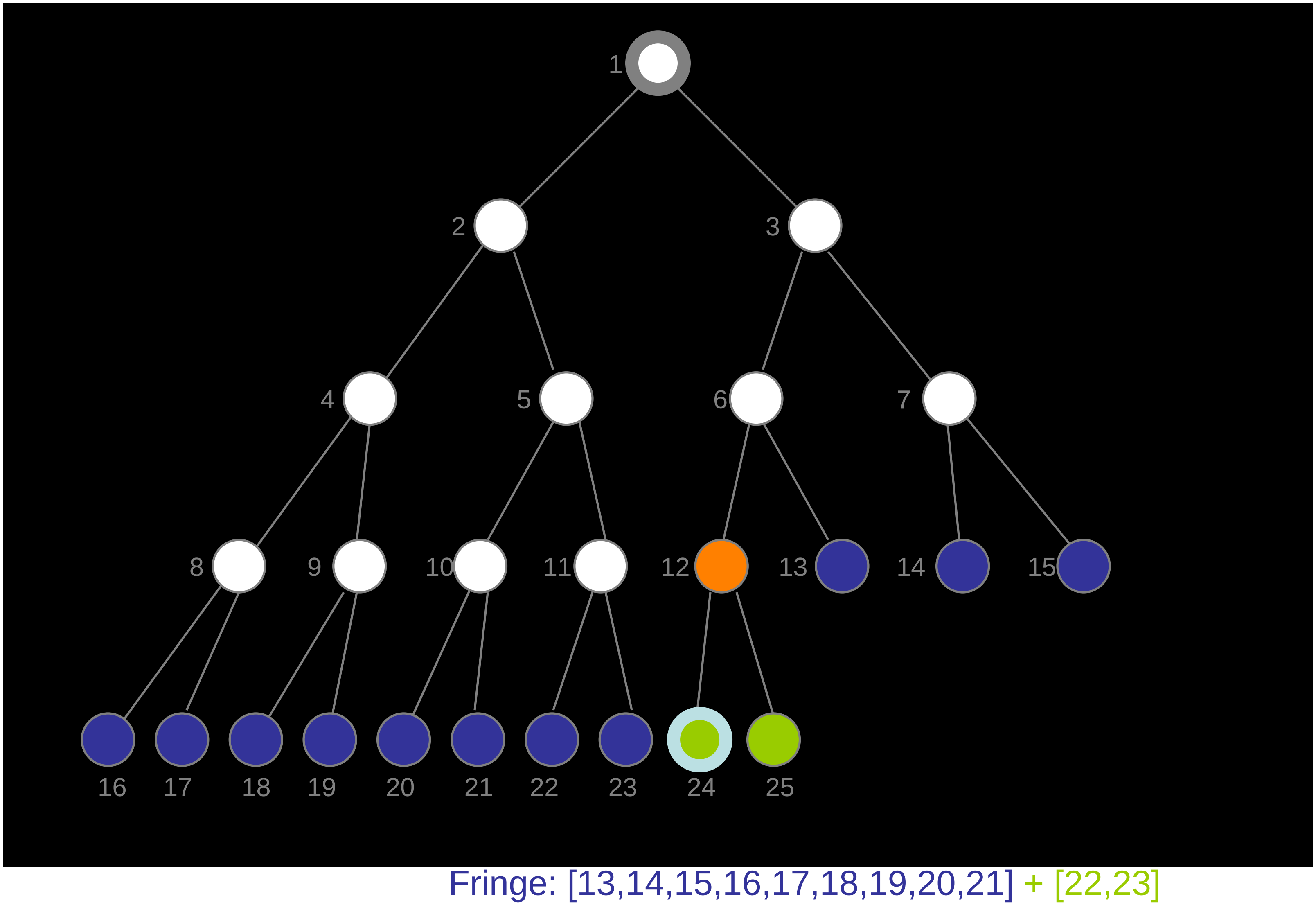
Fringe: [11,12,13,14,15,16,17,18,19] + [20,21]

Breadth-First Snapshot 11



Fringe: [12, 13, 14, 15, 16, 17, 18, 19, 20, 21] + [22,23]

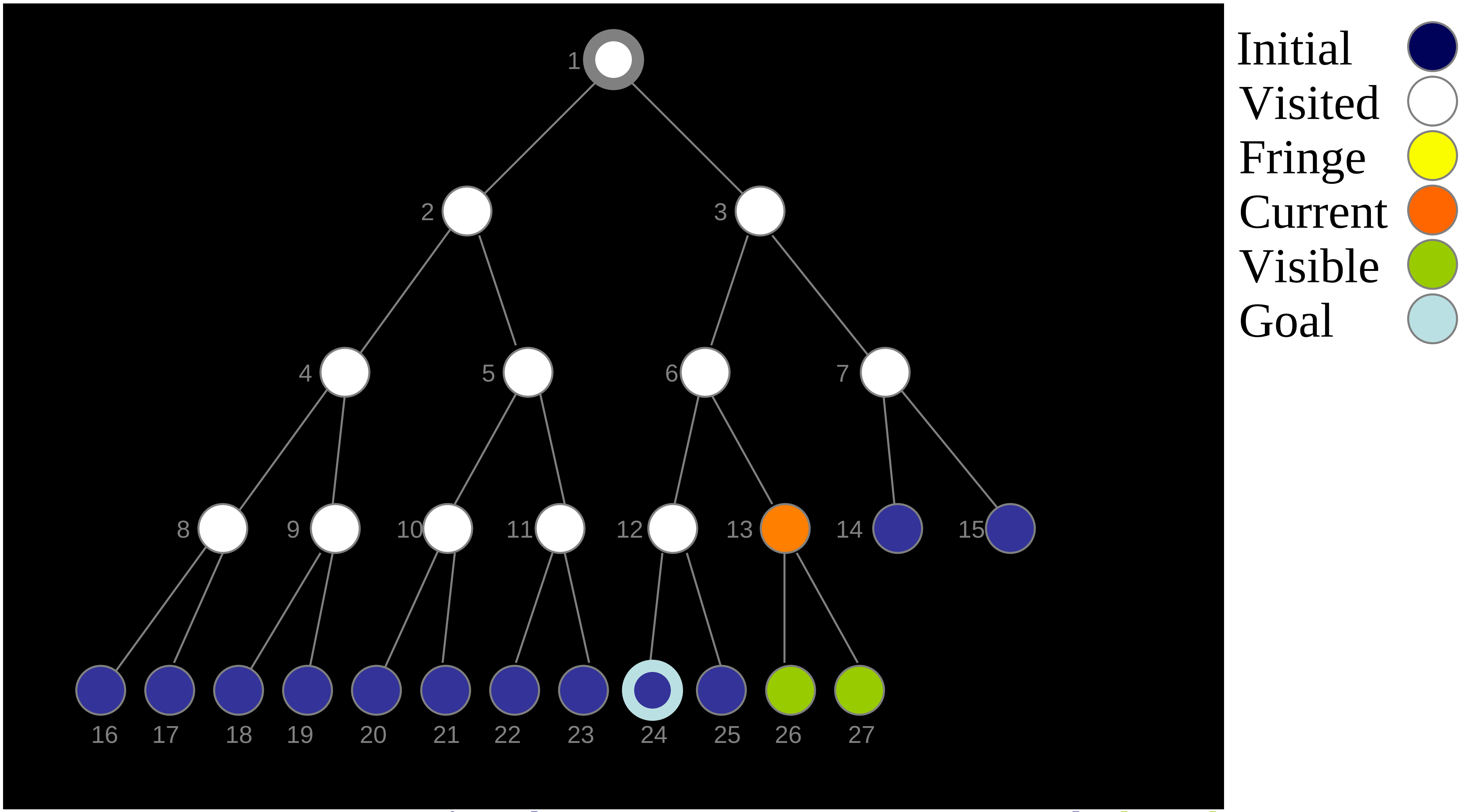
Breadth-First Snapshot 12



Initial
Visited
Fringe
Current
Visible
Goal

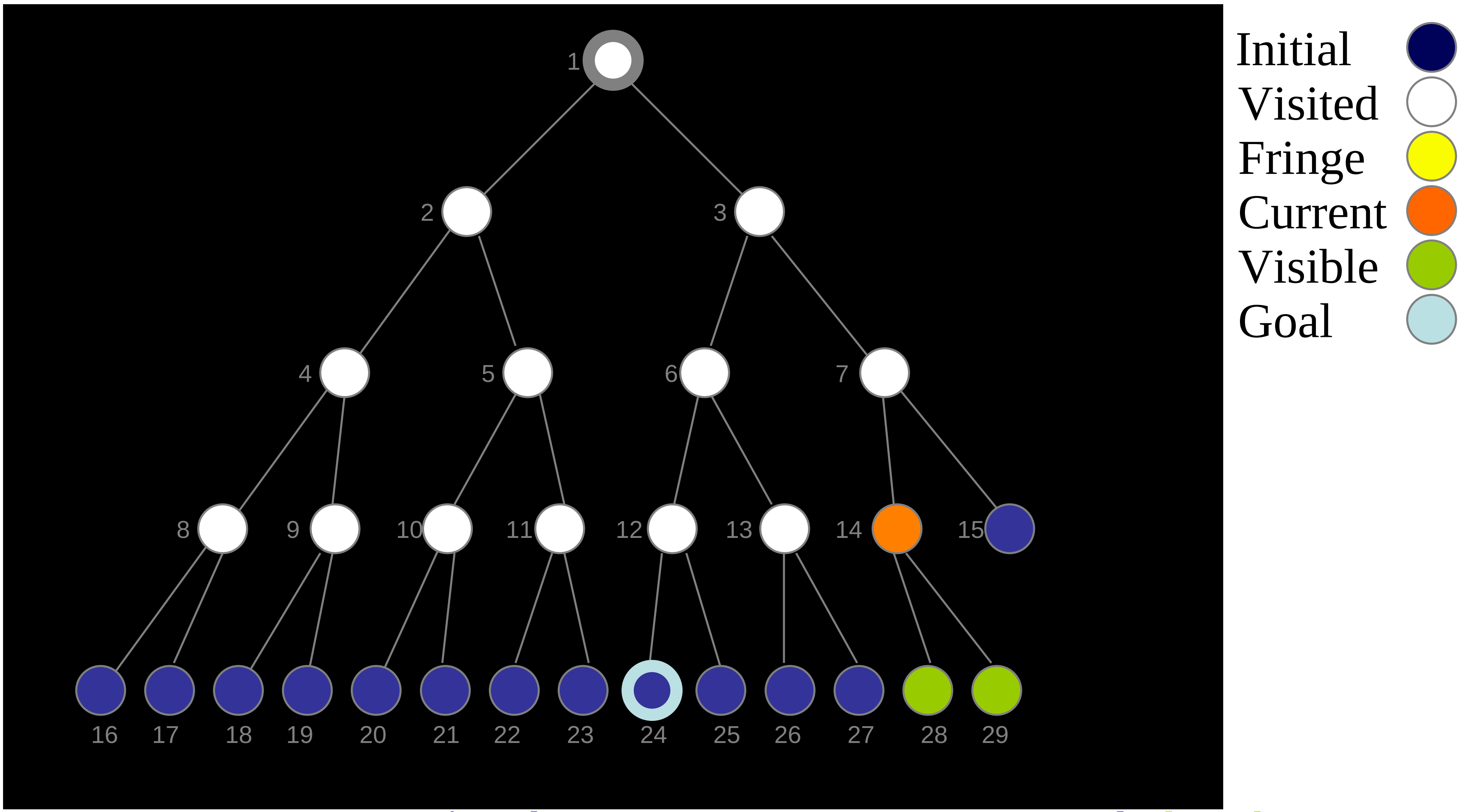
Note:
The goal node is "visible" here, but we can not perform the goal test yet.

Breadth-First Snapshot 13



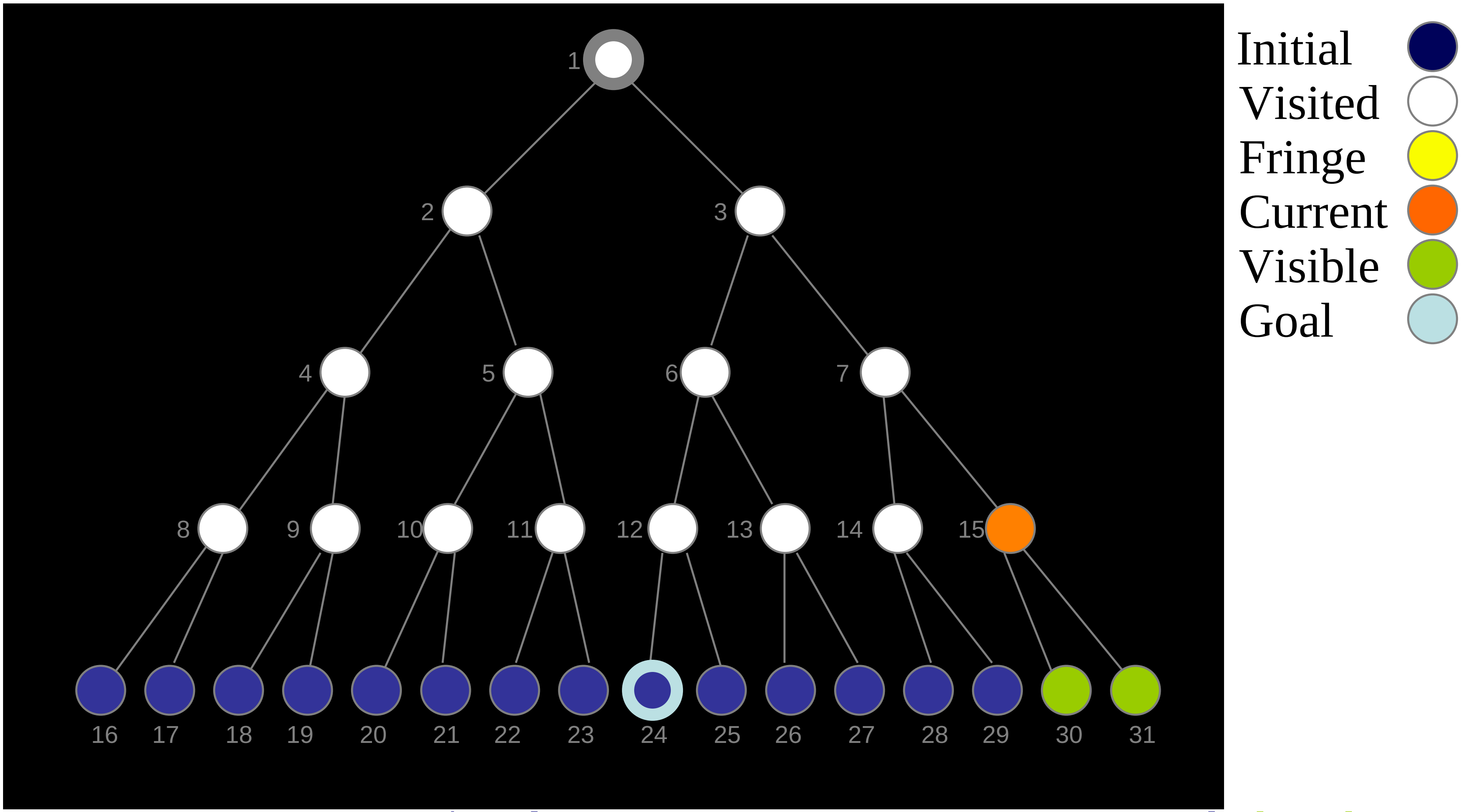
Fringe: [14,15,16,17,18,19,20,21,22,23,24,25] + [26,27]

Breadth-First Snapshot 14



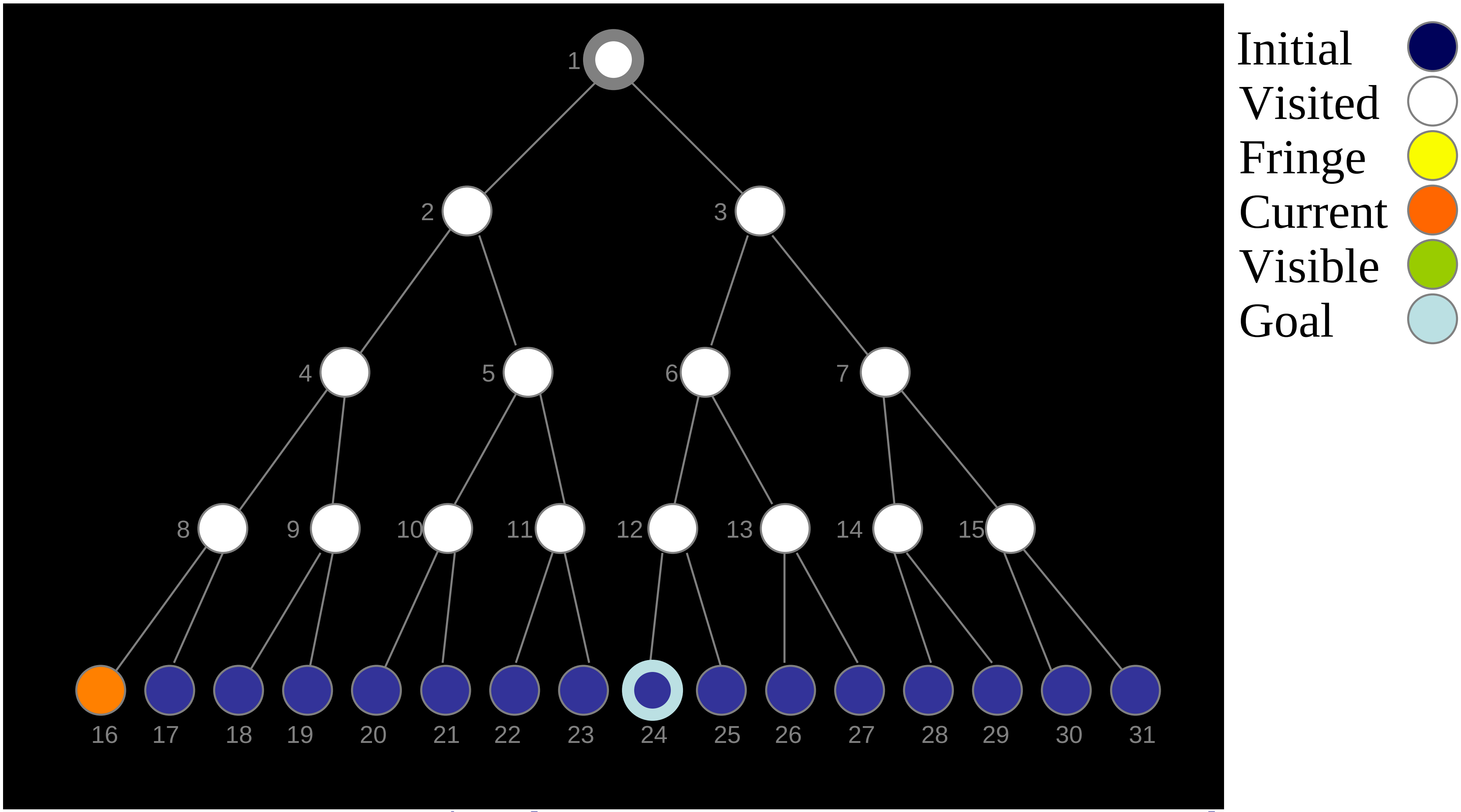
Fringe: [15,16,17,18,19,20,21,22,23,24,25,26,27] + [28,29]

Breadth-First Snapshot 15



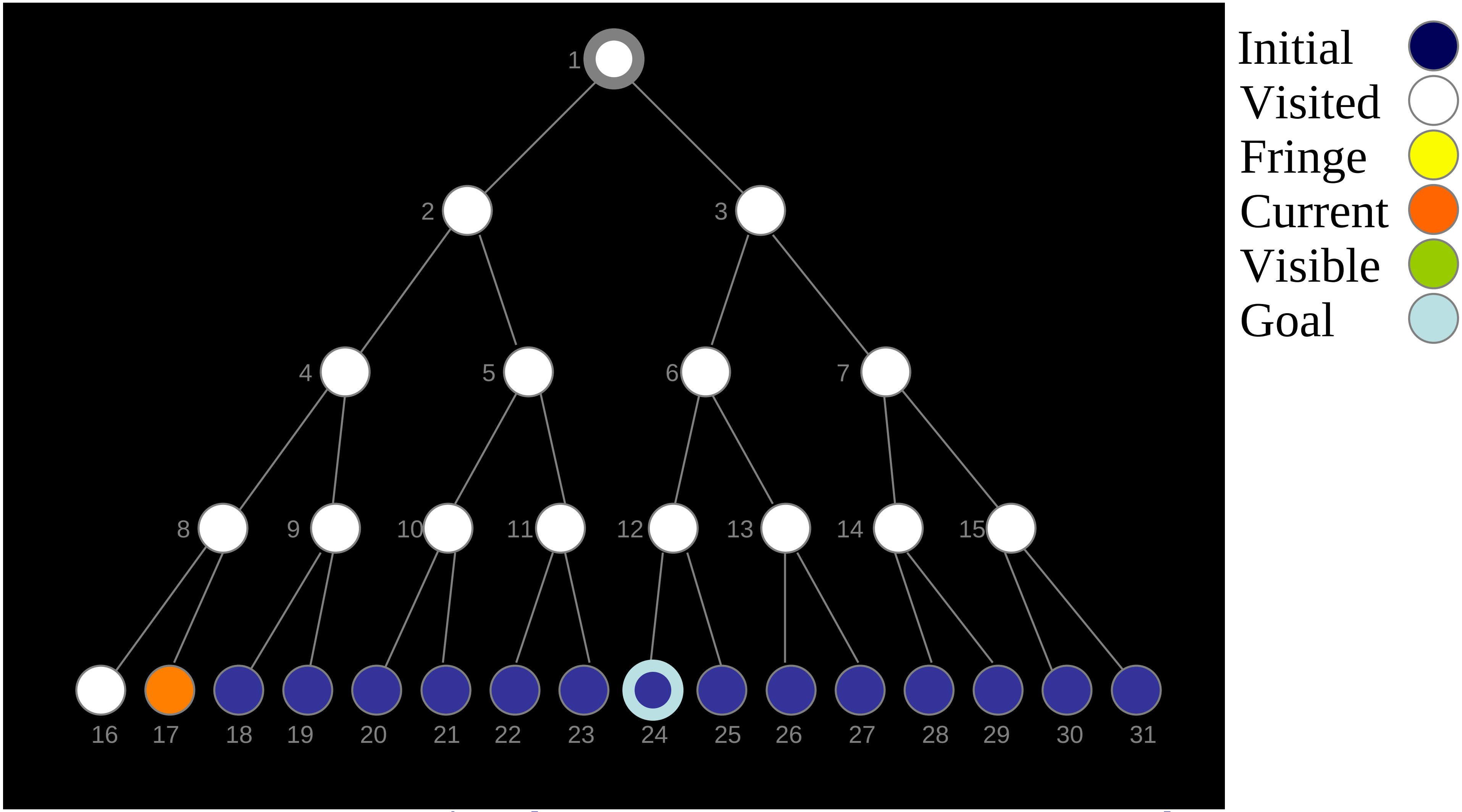
Fringe: [15,16,17,18,19,20,21,22,23,24,25,26,27,28,29] + [30,31]

Breadth-First Snapshot 16



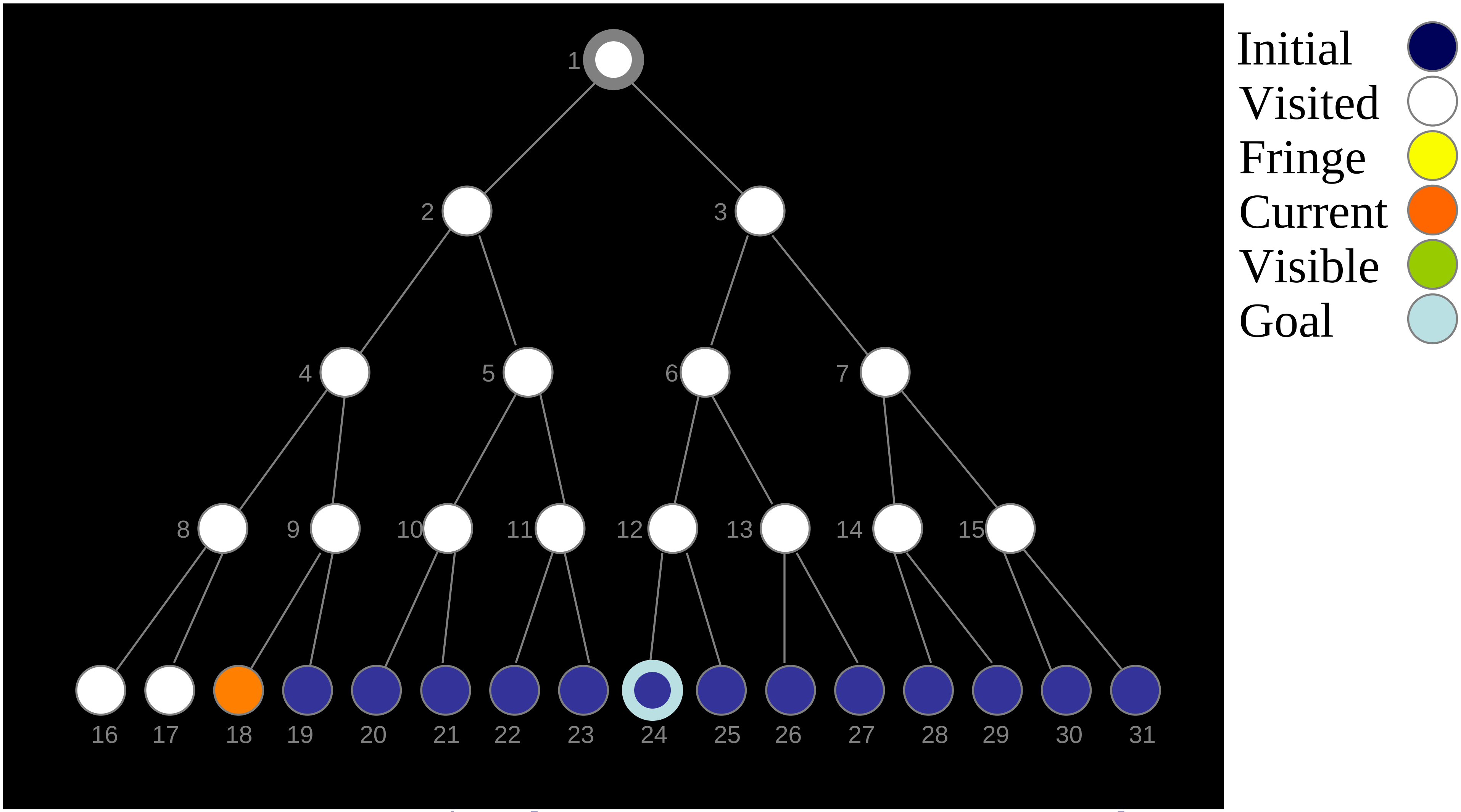
Fringe: [17,18,19,20,21,22,23,24,25,26,27,28,29,30,31]

Breadth-First Snapshot 17



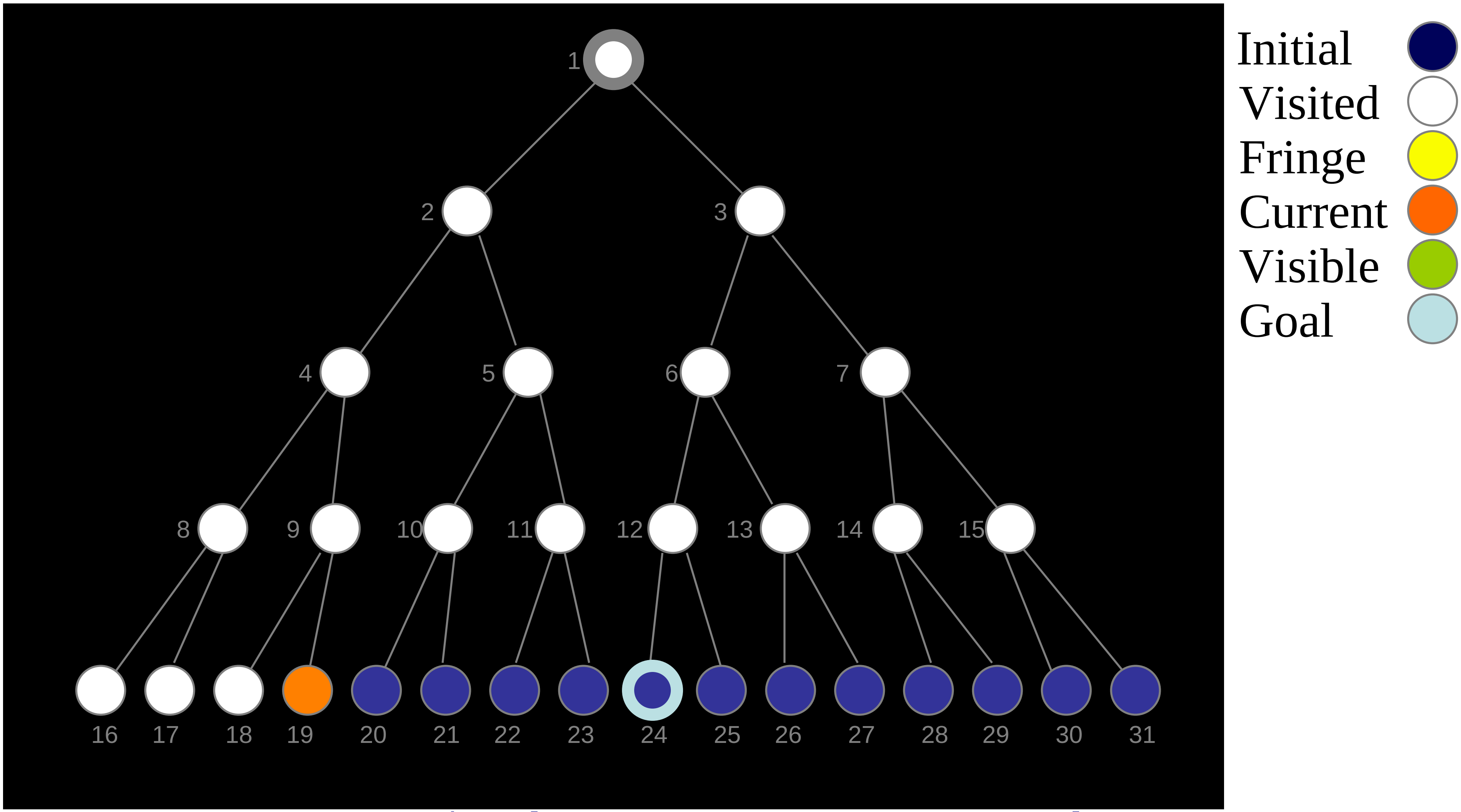
Fringe: [18,19,20,21,22,23,24,25,26,27,28,29,30,31]

Breadth-First Snapshot 18



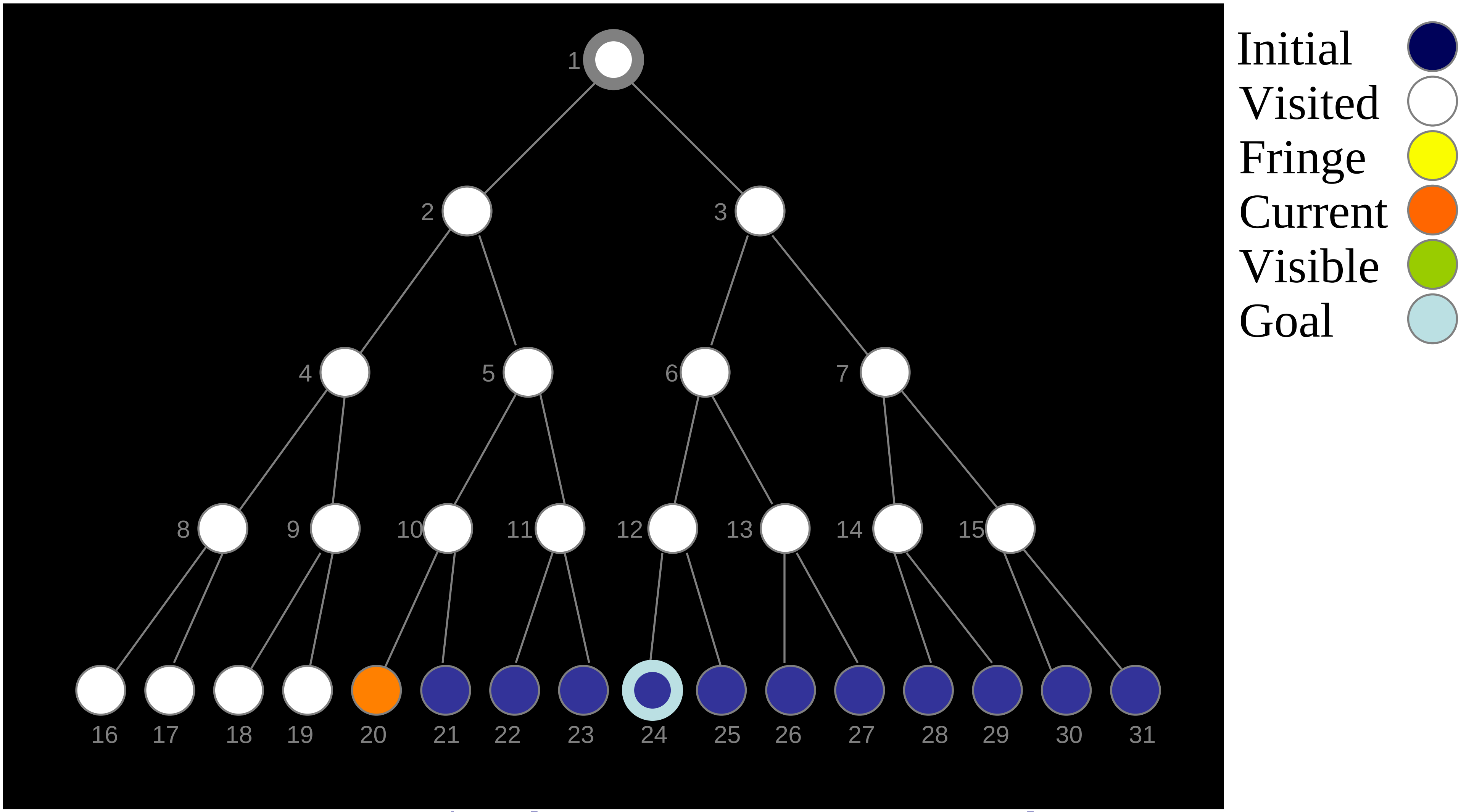
Fringe: [19,20,21,22,23,24,25,26,27,28,29,30,31]

Breadth-First Snapshot 19



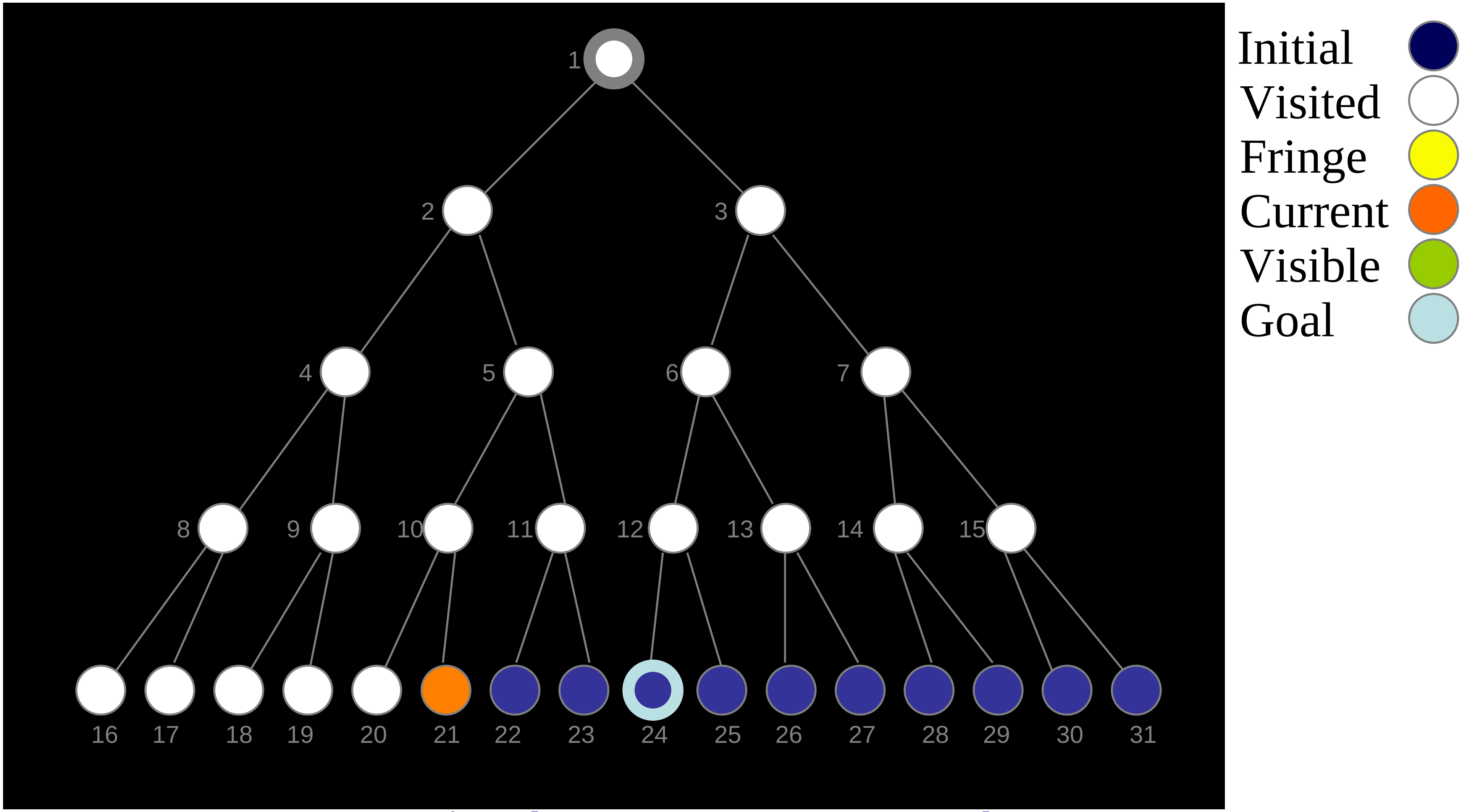
Fringe: [20,21,22,23,24,25,26,27,28,29,30,31]

Breadth-First Snapshot 20



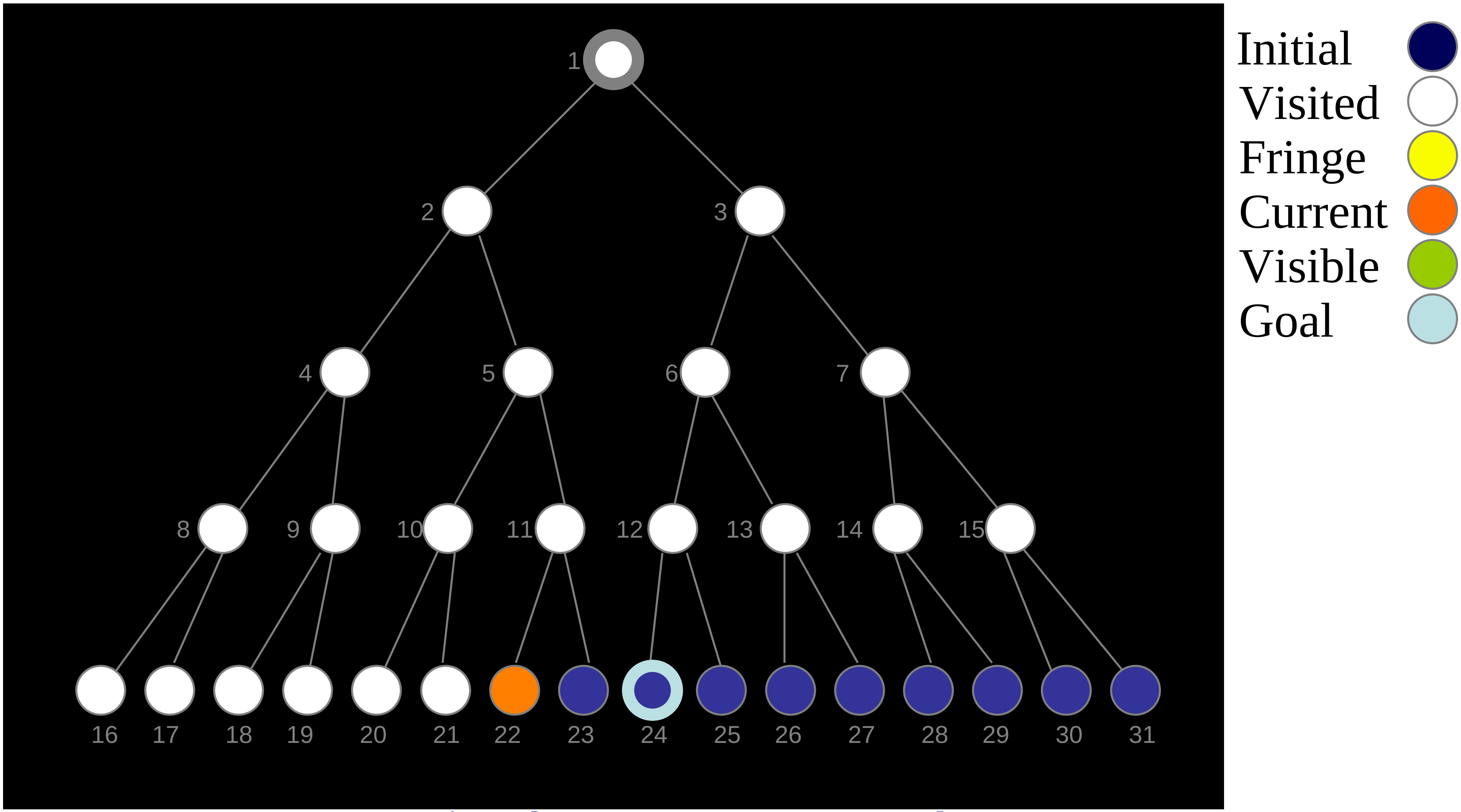
Fringe: [21,22,23,24,25,26,27,28,29,30,31]

Breadth-First Snapshot 21



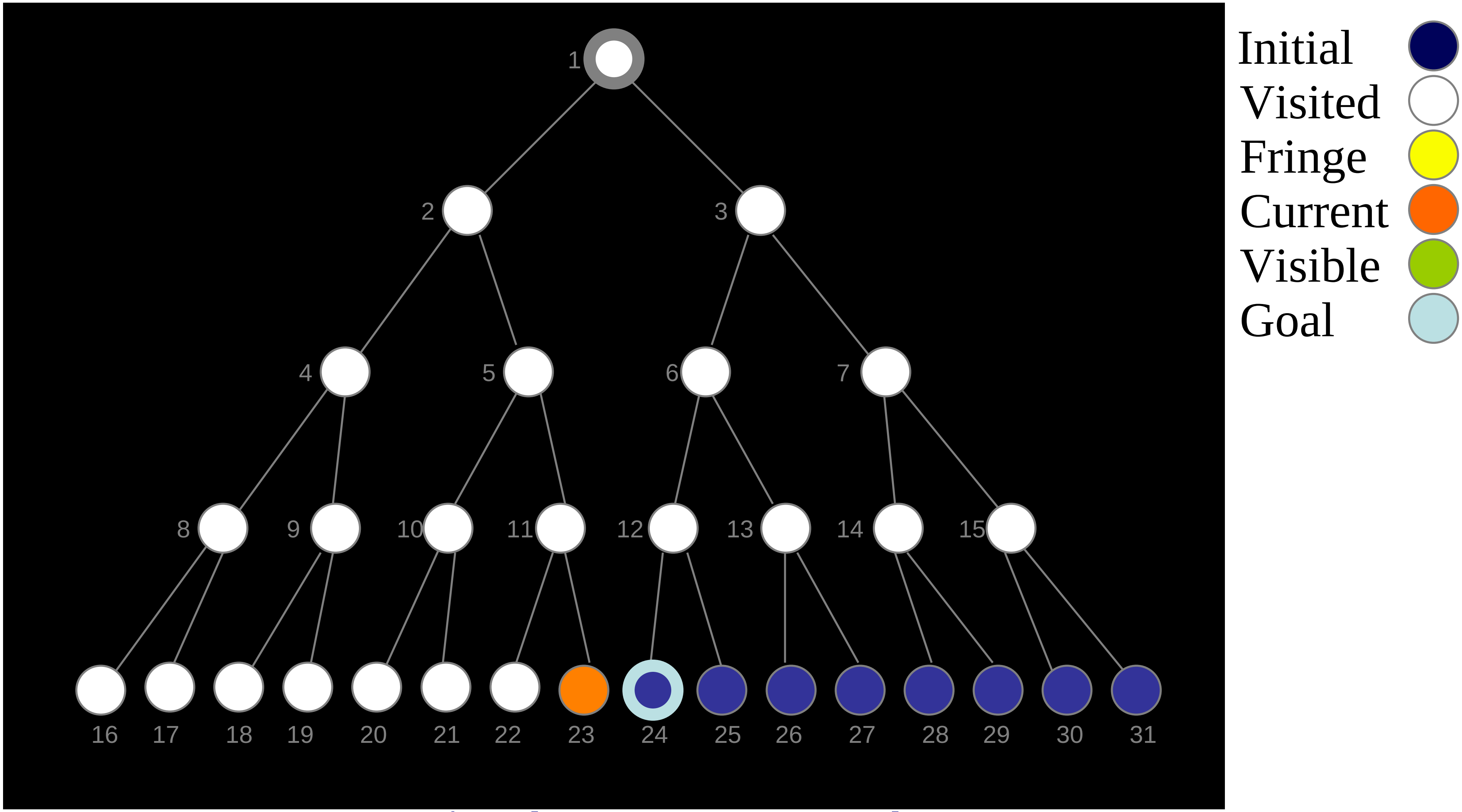
Fringe: [22,23,24,25,26,27,28,29,30,31]

Breadth-First Snapshot 22



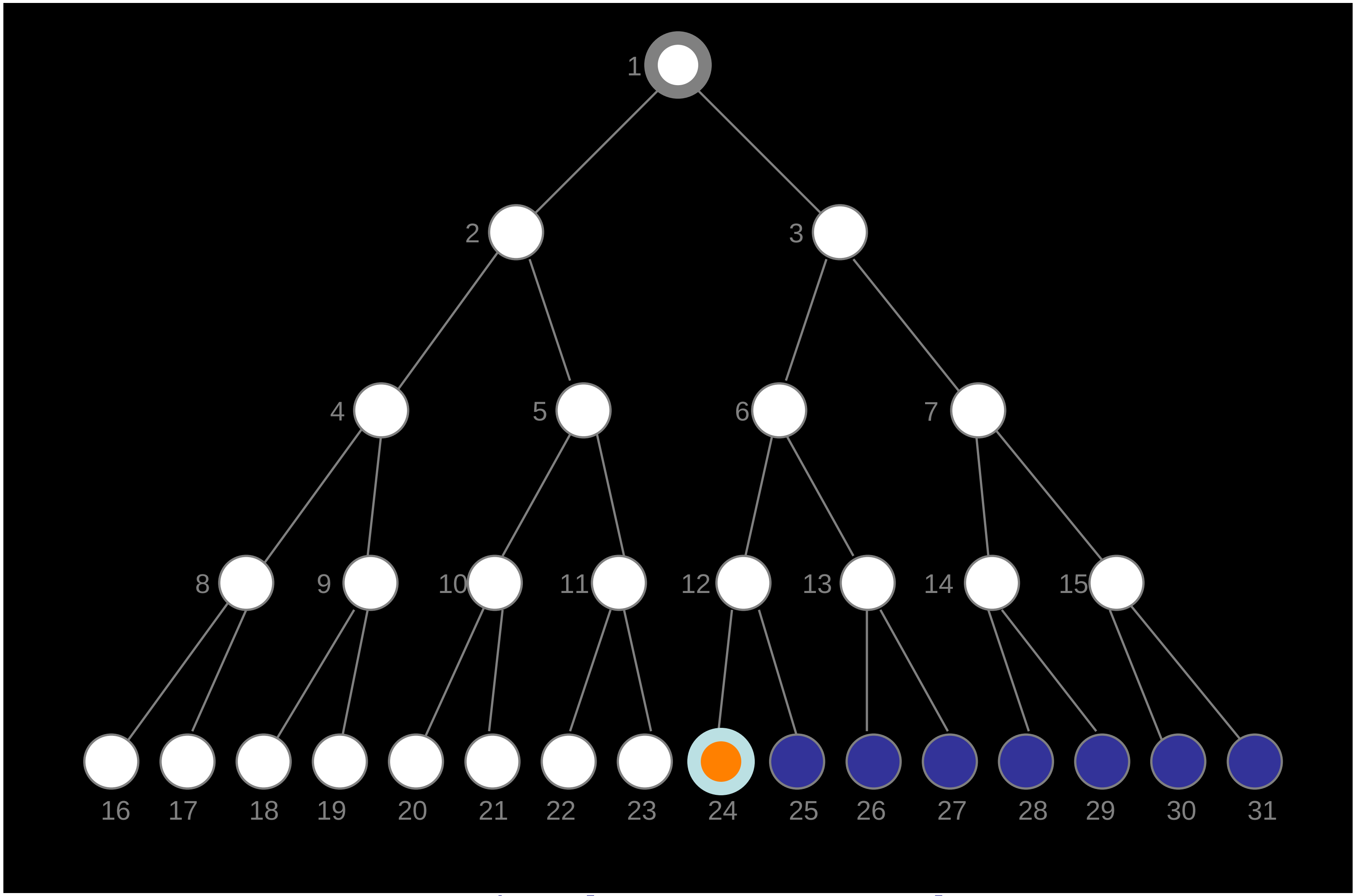
Fringe: [23,24,25,26,27,28,29,30,31]

Breadth-First Snapshot 23



Fringe: [24,25,26,27,28,29,30,31]

Breadth-First Snapshot 24



Initial
Visited
Fringe
Current
Visible
Goal

Note:
The goal test is positive for this node, and a solution is found in 24 steps.

Fringe: [25,26,27,28,29,30,31]

Depth-First

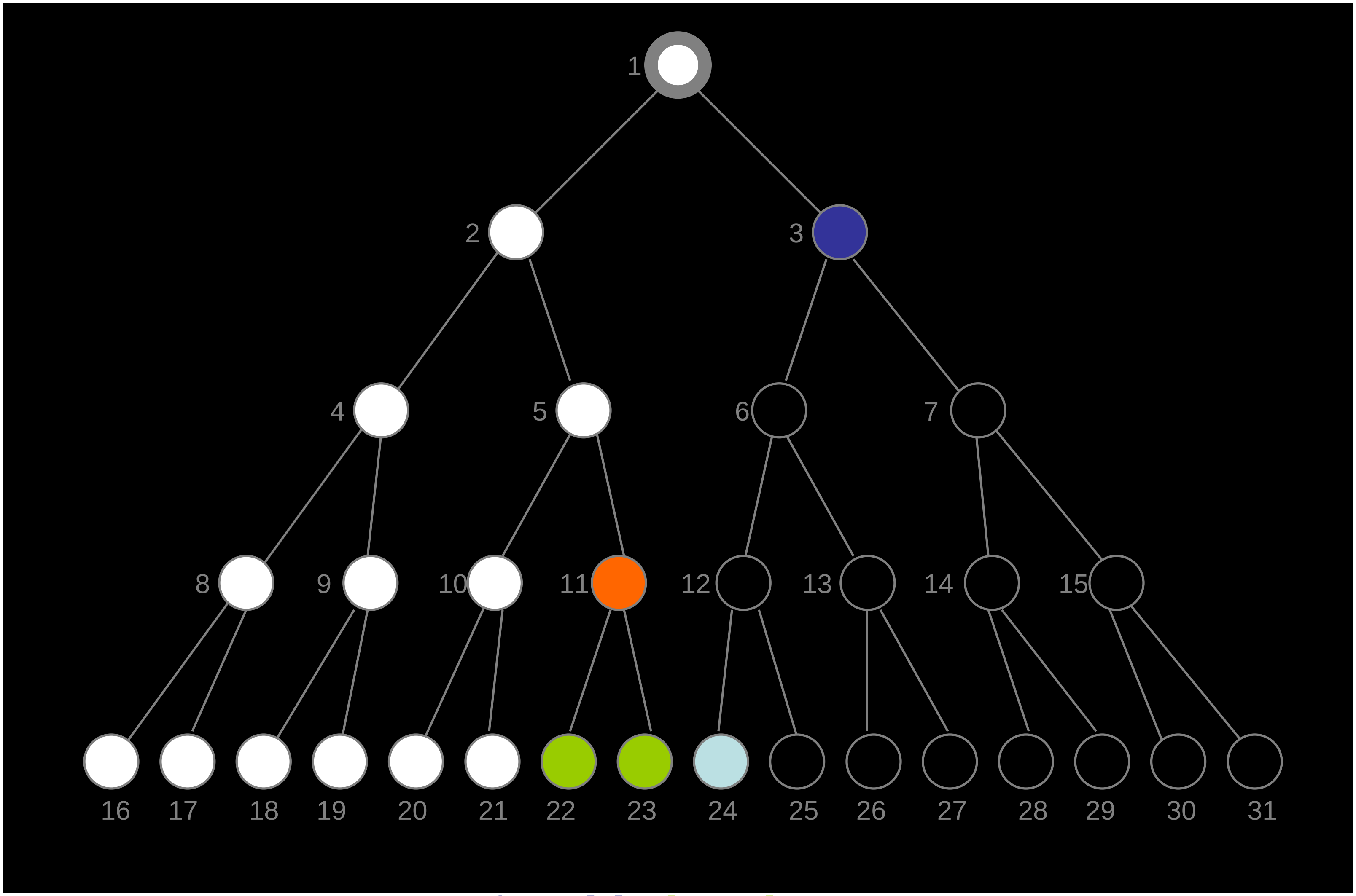
```
function DEPTH-FIRST-SEARCH(problem) returns solution

    return TREE-SEARCH(problem, LIFO-QUEUE())
```

- continues exploring newly generated nodes
  - – achieved by the TREE-SEARCH method by appending newly generated nodes at the beginning of the search queue
    - utilizes a Last-In, First-Out (LIFO) queue, or stack

Time Complexity	b^m
Space Complexity	b*m
Completeness	no
Optimality	no

b	branching factor
m	maximum path length

Depth-First Snapshot



Initial
Visited
Fringe
Current
Visible
Goal

Fringe: [3] + [22,23]

Depth-First vs. Breadth-First

- depth-first goes off into one branch until it reaches a leaf node
  - not good if the goal is on another branch
  - neither complete nor optimal
  - uses much less space than breadth-first
    - much fewer visited nodes to keep track of
    - smaller fringe
- breadth-first is more careful by checking all alternatives
  - complete and optimal
  - very memory-intensive